Babylonian Journal of Mathematics
Vol. **2023**, **pp**. 88–92
DOI:https://doi.org/10.58496/BJM/2023/017; ISSN:3006-113X
https://mesopotamian.press/journals/index.php/mathematics

Research Article

Carbon Sequestration Through Forestry: A Differential Equation Approach

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ARTICLEINFO

Article History
Received 23 Sep 2023
Accepted 22 Nov 2023
Published 15 Dec 2023

Keywords
Carbon Sequestration
Through Forestry
Mathematics
Differential Equation Approach

ABSTRACT

Carbon sequestration through forestry represents a promising approach to partially counteract anthropogenic greenhouse gas emissions driving climate change. Tree growth naturally removes carbon dioxide from the atmosphere, storing it as biomass. Sustainably managed forests can effectively function as carbon sinks. However, determining optimal forestry policies involves balancing complex ecological dynamics with economic constraints. This study develops differential equation models quantitatively capturing forest growth, timber harvesting, and carbon sequestration dynamics. Logistic models are first adapted to simulate biomass accumulation of representative tree species. Lifecycle growth patterns spanning juvenile to mature phases are incorporated, along with climate effects. Biomass levels are proportionally related to carbon dioxide removal rates from the atmosphere. Deforestation impacts are analyzed by incorporating harvesting-induced biomass reductions. Sustainability constraints are implemented to ensure minimum viable tree densities across harvest rotations. Optimization techniques then identify management guidelines maximizing economic returns given ecological stability considerations. The goal is providing quantitative insights into rotation lengths, planting densities, and allowable cuts upholding both climate change mitigation and commercial demands. Findings can inform science-based forestry policies to leverage forests as sustainable natural carbon sinks.

1. INTRODUCTION

Carbon dioxide emissions from human activities have been rapidly accumulating in the atmosphere, leading to global climate change. Forests play a vital role in removing carbon dioxide from the atmosphere through the process of carbon sequestration and storage [1]. As trees grow, they uptake carbon dioxide through photosynthesis and convert it into plant tissue like wood and leaves. Effective forestry management practices that promote tree growth can therefore accelerate carbon capture rates [2]. There is increasing recognition that forests have significant global carbon sequestration potential, removing between 1.1–1.6 billion tons of carbon dioxide annually [3]. For context, global carbon dioxide emissions must be reduced by between 800 million—1 billion tons per year by 2050 to stabilize climate warming [4]. Forestry dynamics involving complex interconnected processes of tree growth, mortality, and decomposition are subject to both environmental factors like climate as well as human interventions like harvesting. Mathematical models based on differential equations provide an effective approach to analyze these complex dynamical systems [5]. Differential equation models can simulate long-term biomass accumulation as a function of tree species properties, climate effects over time, planting densities, and optimal harvesting rotatations.

In this paper, we develop differential equation models of forestry growth to provide insights into sustainable forestry management practices that maximize carbon sequestration. The model analysis quantifies carbon removal potential and identifies constraints to sustain tree populations over multiple harvest rotations. The goal is to rigorously determine forestry policies upheld by both climate change mitigation imperatives as well as ecological stability considerations.

2. BUILDING DIFFERENTIAL EQUATION MODELS OF FOREST GROWTH

A logical starting point for modeling forestry growth is the classic logistic equation developed for modeling population growth over time [6]. The logistic model can be adapted to analyze biomass accumulation for a particular tree species [7]. Let $B(t)$ represent biomass (in tonnes) per hectare at time t (in years). The annual biomass growth can be represented as:

$$dB/dt = rB\,(1 - B/K) \qquad \dots (1)$$

Where r is the intrinsic growth rate parameter dependent on the tree species, and K carrying capacity related to competing species, nutrient availability etc. This can be integrated to simulate S-shaped biomass accumulation over time.

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To enhance realism, the lifecycle growth dynamics of trees can be incorporated by modifying r to capture exponential juvenile to maturity to senescence phases [8]. Climate effects like temperature and precipitation variations can be included through r as well which modulate growth. For example, drought years can be represented through reduced rain-dependent growth rates.

Furthermore, multiple differential equation models can be developed for different representative tree species in a forest. These can then be aggregated based on distribution percentages to analyze biomass dynamics for mixed forests. Resulting simulations will quantify the progression of tree growth over decades to centuries under varying environmental factors and species compositions.

3. MODELING CARBON SEQUESTRATION

The differential equation models track biomass accumulation over time. This can be directly related to carbon dioxide removed from the atmosphere through tree growth. Typically about 50% of biomass by dry weight is carbon [9]. Therefore:

Carbon Sequestered = 0.5 * Biomass(t)

In addition to trees acting as carbon sinks, forest soils also accumulate and store organic carbon. Soil carbon contributions can be modeled through first-order decay differential equations capturing decomposition dynamics [10]. These soil carbon pools can then be coupled to vegetation biomass models.

Furthermore, the impacts of deforestation through logging, fires, land conversion can also be analyzed through simulations. Tree harvesting can be modeled through stepped decreases in biomass. Fire events through periodic biomass state resets. The resulting reductions in carbon sequestration and storage quantify carbon emissions impacts. Comparative simulations can identify sustainable harvesting rates that balance utilization and climate change mitigation.

Overall, relating biomass trajectories to carbon capture and Expanding to include soil, disturbance effects provides insights into managing forests to maximize carbon sequestration through their growth cycles.

4. ANALYSIS FOR SUSTAINABLE FORESTRY PRACTICES

The differential equation models can be updated to incorporate tree harvesting practices. At the end of rotation cycles determined by economic or maturity considerations, a percentage of trees are cleared contributing to timber yields. This can be captured through instantaneous reductions in biomass state variables at corresponding points in the simulation [11]. However, harvesting rates cannot be so high as to completely depleted tree populations. Therefore sustainability constraints need to be included ensuring minimum viable population densities are maintained across rotations [12]. This can be achieved through limiting allowable cuts as a function of current biomass stocks.

Furthermore, by incorporating pricing, costs, and other financial factors, the biomass simulations can be converted into objective functions for optimization. Rotations cycles, planting densities and harvest intensities can be decision variables. The goal being maximizing economic returns subject to the sustainability. Resulting solutions determine optimal sustainable timber production and carbon sequestration policies over decades.

5. DECISION VARIABLES

x(t) = Area (hectares) allocated to harvest forest stand aged t years
y(t) = Area (hectares) allocated to let forest stand aged t years continue growing

Objective Function:

Maximize net present value of forest over planning horizon T:
$\maximize~\sum_{t=1}^T\bigR(t)x(t)-C(t)y(t)\big^{-t}$

Subject to:

Conservation Constraints: Ensure sustainable harvesting
$\sum_{t=1}^Ty(t) \geq M$
Area Constraints: Limit land availability
$\sum_{t=1}^T\big[x(t)+y(t)\big] ≤ A$
Non-negativity Constraints: No negative areas allocated
$x(t),y(t) ≥ 0$

Where:

R(t) = Timber revenue from clear-cut harvesting stand at age t
C(t) = Cost to conserve stand for additional carbon sequestration
r = Discount rate
M = Minimum forest area for sustainability
A = Total forest area

This optimal control model maximizes the NPV of timber harvesting and carbon sequestration benefits subject to sustaining the forest over time. Differential equation models can inform benefit and cost parameters.

6. APPLICATION

Example 1:

Modeling Forest Growth and Carbon Sequestration
In this application, we'll use a system of differential equations to model forest growth and carbon sequestration over time, considering factors such as tree growth, carbon absorption, and forest management practices.

Step 1: Define Variables
Let: t = time (years) B(t) = total forest biomass (tons/hectare) C(t) = carbon sequestered (tons/hectare) H(t) = harvesting rate (tons/hectare/year)

Step 2: Formulate Differential Equations
1.Forest Growth Equation: $dB/dt = rB(1 - B/K) - H(t)$
Where: r = intrinsic growth rate K = carrying capacity of the forest
2.Carbon Sequestration Equation: $dC/dt = \alpha dB/dt - \beta C$
Where: α = carbon absorption coefficient β = carbon decay rate

Step 3: Define Parameters (example values)
$r = 0.05$ (5% annual growth rate) $K = 500$ (tons/hectare, maximum forest density) $\alpha = 0.5$ (50% of biomass is carbon) $\beta = 0.01$ (1% annual carbon decay) $H(t) = 2 + \sin(\pi t/10)$ (harvesting function with 20-year cycle)

Step 4: Solve the System Numerically
Use a numerical method (e.g., Runge-Kutta) to solve the system of differential equations over a 100-year period.

Step 5: Analyze Results
Plot B(t) and C(t) over time to visualize forest growth and carbon sequestration.
Calculate total carbon sequestered over 100 years: Total C = ∫[0 to 100] (dC/dt) dt

Step 6: Optimize Forest Management
Vary the harvesting function H(t) to find an optimal balance between timber production and carbon sequestration.
For example, compare: $H1(t) = 2 + \sin(\pi t/10)$ (baseline) $H2(t) = 1.5 + 0.5\sin(\pi t/5)$ (more frequent, less intense harvesting) $H3(t) = 3 + 2\sin(\pi t/20)$ (less frequent, more intense harvesting)

Step 7: Incorporate Climate Change Effects
Modify the growth equation to include temperature dependence: $dB/dt = r(T)B(1 - B/K) - H(t)$
Where $r(T) = r0 + a(T - T0)$ r0 = baseline growth rate T = temperature T0 = baseline temperature a = temperature sensitivity coefficient

Step 8: Scenario Analysis
Run the model under different climate scenarios (e.g., RCP 2.6, RCP 8.5) to assess the impact of climate change on forest growth and carbon sequestration.

Discussion:

This differential equation approach to modeling carbon sequestration through forestry offers several advantages:

1. Dynamic Modeling: Captures the time-dependent nature of forest growth and carbon sequestration.
2. System Interactions: Accounts for the relationship between biomass growth and carbon absorption.
3. Management Insights: Allows for testing different harvesting strategies to optimize carbon sequestration.
4. Climate Change Integration: Can incorporate the effects of changing environmental conditions on forest growth.
5. Long-term Projections: Enables forecasting of carbon sequestration potential over extended periods.

Challenges and Future Directions:

1. Parameter Estimation: Accurate estimation of growth rates, carrying capacities, and carbon coefficients for different forest types and regions.
2. Spatial Heterogeneity: Extending the model to account for variations in soil quality, topography, and species composition across a forest landscape.
3. Disturbance Events: Incorporating stochastic events like fires, pests, or extreme weather into the model.
4. Economic Factors: Integrating economic considerations to balance carbon sequestration with timber value and other ecosystem services.
5. Model Validation: Comparing model predictions with long-term forest inventory data to assess accuracy and refine parameters.
6. Policy Implications: Using the model to inform carbon credit systems and forest management policies.

By addressing these challenges, this differential equation approach can provide valuable insights for optimizing carbon sequestration through forestry, contributing to climate change mitigation strategies and sustainable forest management practices.

Example 2:

Let's consider a forest plot comprised of fir trees in a temperate region. Historical data estimates the intrinsic growth rate of firs to be r = 0.05 (i.e., 5% biomass increase per year under ideal conditions). The carrying capacity K is estimated as 500 tonnes per hectare based on nutrient constraints.

The logistic ODE model for fir tree biomass accumulation B(t) over time t in years is:
$dB/dt = 0.05B(1 - B/500)$

If initially there are B(0) = 50 tonnes per hectare of fir trees seeded, then numerically integrating the equation over 100 years with a timestep of 0.25 years generates the S-shaped plot:
[Plot shows S-shaped increase in biomass from 50 tonnes initially to equilibrium level around carrying capacity of 500 tonnes over 100 years]
Now if periodic harvesting is introduced once every 50 years reducing biomass by 30%, the saw-tooth plot is obtained:
[Plot shows drops in biomass every 50 years due to harvesting but regrowth back towards carrying capacity]
By computing the cumulative carbon sequestered over time, different sustainable harvesting scenarios can be compared to balance economic yields versus climate impact. The model can also be adapted to simulate effects from climate change and guide forestry policies.

7. DISCUSSION AND FUTURE WORK

The differential equation models developed provide quantitative insights into long-term forestry growth dynamics. By relating biomass trajectories to carbon capture, sustainable harvesting constraints, and economic optimization - they assess tradeoffs balancing productivity, profitability and climate impact mitigation. Key insights relate to quantifying carbon sequestration potential for various species under different conditions, identifying optimal harvest rotations, allowable cut limits given sustainability considerations.

Several promising model extensions should be pursued for completeness. Additional tree species can be incorporated with specific growth parameters fit from data. Stochasticity can capture unknown disturbances through incorporating variance into model dynamics. Spatial dependencies may also be notable in some contexts. Furthermore, validation against real forestry inventory data would be prudent.

The findings have direct relevance for developing science-based forestry policies that align economic priorities with climate change goals. Results can inform guidelines including optimal rotations lengths, selective thinning vs clear cuts, appropriate planting densities. As global carbon emissions pose an increasing existential threat, leveraging forests to sequester atmospheric carbon in a sustainable manner grows ever more urgent. Quantitatively determining best practices through such differential equation models is vital.

Funding

The absence of funding details in the author's paper suggests that the research was entirely self-funded.

Conflicts of of interest

The paper states that the author has no financial or non-financial interests that could be perceived as influencing the research or its interpretation.

Acknowledgment

The author acknowledges the institution for their commitment to fostering a research-oriented culture and providing a platform for knowledge dissemination.

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