Research Article

On commutativity of alternative rings with [xy^n x±yx^n y,x]=0

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ARTICLEINFO

ABSTRACT

Let R be a n-torsion free with identity 1 ,In this article we investigate and prove the commutativity of alternative ring of the property(p_1), (p_2) and(p_3) under suitable constraints.

(p_1)   [xy^n x±yx^n y,x]=0

(p_2)   [xy^n x±yx^n y,y]=0

(p_3)   [x(⟦xy)⟧ ^2+(⟦xy)⟧ ^2 x,x] = 0. $\forall$ x,y $\in$ R.

1. INTRODUCTION

In this paper R represent an alternative ring with center $Z(R) = \{x \in R \mid xy = yx\}$, the commutator $[x,y] = xy - yx$ and anti-commutator $xy \, o \, xy = xy + yx$ for any pair element $x,y \in R$, for any positive integer n, an element $x \in R$ is said to be $n! -$torsion free if and only If $nx = 0$ implies $x = 0$. The associator (x,y,z) is define by $(x,y,z) = (xy)z - x(yz)$ for all $x,y,z \in R$,This play a key role in the study of non-associative rings. It can be viewed as a measure of the non-associativity of a ring. In terms of associator, a ring is called left alternative if $(x,x,y) = 0$ right alternative if $(y,x,x) = 0$ for all $x,y \in R$ and alternative if both condition hold. i.e.$(x,y,y) = 0$ and $(y,y,x) = 0$.

In [2] established that a division ring R is commutative if and only if $[xy,yx] = 0$. Also generalize Guptar's result which assert that a semi prime ring R in which $[xy,yx] = xy^2x - yx^2y \in Z(R)$ or $xy \, o \, xy = xy^2x + yx^2y \in Z(R)$ is necessary commutative, also [1] proved the commutativity of associative ring satisfies the identity $(xy)^2 = xy^2x$. also in their paper proved the properties : $xy \, o \, xy = xy^2x + yx^2y \in Z(R)$ and $xy^nx \pm yx^ny \in Z(R)$,most be commutative In addition, [3] have established the commutativity of non-associative ring satisfying the identities $(xy)^2 = x^2y^2$ and $(xy)^2 \in Z(R) \forall x,y \in R$.

Further, [4] established the commutativity of non-associative primitive rings satisfying the identities: $x(x^2 + y^2)+(x^2 + y^2)x \in Z(R)$ and $x(xy)^2 - (xy)^2x \in Z(R)$. Recently [5] show that some results on commutativity of some 2-torsion free non associative rings with unity satisfy:

$(\alpha\beta)^2 - \alpha\beta \in Z(R) \; for \; all \; \alpha\beta \; in \; R$ Motivated by these observation it is natural to look commutativity of alternative rings satisfies

Motivated from them we establish the commutativity of ring with condition $(p_1),(p_2)$ and commutativity of alternative rings with (p_3) with suitable constraint.

Main Result.

The following are the main Results.

Theorem 2.1.

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Suppose that a n-torsion free an alternative ring R with identity 1 and there exist a positive integer n such that:
$[xy^n x \pm yx^n y, x] = 0$ or $[xy^n x \pm yx^n y, y] = 0$ $\forall\, x, y \in R$. Then R is commutative.

Proof.

By hypothesis, we have
$[xy^n x \pm yx^n y, y] = 0$. In this property we consider $[xy^n x + yx^n y, y] = 0$
$[yxy^n x - xy^n xy + y^2 x^n y - yx^n y^2] = 0$
$[y, (xy^n x)] + [y, (x^n y - yx^n)y] <=> [y, xy^n x] + y[x^n, y]y = 0$
Replace $y = y + 1$ in 2.1 above and applied n-torsion free we obtained
$[x\,^n C_1 y_1 x, y] + [x\,^n C_2 y_2 x, y] + \ldots\ldots\ldots\ldots\ldots\ldots + [x\,^n C_n y_n x, y]$
$x[\,^n C_1 y_1 + \,^n C_2 y_2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + \,^n C_n y_n]x = [x^n, y^2] + yx^n = 0$
We used binomial expansion and by inserting $y = y + 1$ for n-times and using the previous we obtained identity in every stage in above we had
$x[(\,^{n-1} C_{n-1}\,^{n-1} C_{n-2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + \,^n C_n)x, y]$.
This gives,
$n![x^2, y] = 0$ in view of n-torsion free condition, we get on the commutator $[x^2, y] = 0$ for $n > 2$ then R is commutative.

Remark 2.2: The following Corollary is an immediate consequence of our main result if we set $n = 2$.

Corollary 2.3

Let R be a 2-torsion free an alternative ring with identity 1. If R has a properties:
$[xy^2 x \pm yx^2 y, x] = 0$ or $[xy^2 x \pm yx^2 y, y] = 0$ $\forall\, x, y \in R$. Then R is commutative ring.

Proof

Since $[xy \, o \, xy, y]$, now we consider $[xy^2 x - yx^2 y, y] = 0$.
$[xy^2 x - yx^2 y, y] = 0$
$[yxy^2 x - xy^2 xy + yx^2 y^2 - y^2 x^2 y] = 0$
$[y, xy^2 x] + y[x^2, y]y = 0$ 2.3
Replace $y = y + 1$ in 2.3 above and applied 2-torsion free we obtained
$y^2 x^2 - x^2 y^2 = 0 <=> [y^2, x^2] = 0$ 2.4
Replace $y = y + 1$ in 2.4 above we had
$yx^2 - x^2 y = 0 <=> [y, x^2] = 0$ 2.5
Replace $y = y + 1$ in 2.5 above and applied 2-torsion free we obtained
$yx - xy = 0 <=> [y, x] = 0$ or $yx = xy$ implies R is commutative.

Theorem 3.1

Let R be a 2,3-torsion free alternative ring with unity satisfy $[x(xy)^2 + (xy)^2 x, x] = 0$, Then R is commutative.

Proof

From the hypothesis above in 3.1
$x[x(xy)^2 + (xy)^2 x] = [x(xy)^2 + (xy)^2 x]x$,   for all x, y $\in$ R.   3.2
Substitute $x = (1 + x)$ in (3.2), apply 2,3 torsion free and use (1) we get
$y^2 x = xy^2$,   for all x, y $\in$ R.   3.3
Substitute $y = (y + 1)$ in (3.3) and Apply 2-torsion
This implies $xy = yx$ and R is commutative.
Since R is a commutative ring and satisfies the identities either $(xx)y = x(xy)$ or
$y(xx) = (yx)x$, so that R is an alternative ring. Hence an alternative ring R with identity together with commutativity yields $(x, x, y) = 0 = (y, x, x)$, which completes the proof.

References

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