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Research Article

On commutativity of alternative rings with $[xy^nx \pm yx^ny, x] = 0$

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ABSTRACT

Let R be a n-torsion free with identity 1, In this article we investigate and prove the commutativity of alternative ring of the property (p_1), (p_2) and (p_3) under suitable constraints.

(p_1) $[xy^nx \pm yx^ny, x] = 0$
(p_2) $[xy^nx \pm yx^ny, y] = 0$
(p_3) $[x(xy)^2 + (xy)^2x, x] = 0.\ \forall x, y \in R.$

1. INTRODUCTION

In this paper R represent an alternative ring with center $Z(R) = \{x \in R\ |xy = yx\}$, the commutator $[x, y] = xy - yx$ and anti-commutator $xy\ o\ xy = xy + yx$ for any pair element $x, y \in R$, for any positive integer n, an element $x \in R$ is said to be $n!$ −torsion free if and only If $nx = 0$ implies $x = 0$. The associator (x, y, z) is define by $(x, y, z) = (xy)z - x(yz)$ for all $x, y, z\ \epsilon\ , this$ plays a key role in the study of non-associative rings. It can be viewed as a measure of the non-associativity of a ring. In terms of associator, a ring is called left alternative if $(x, x, y) = 0$ right alternative if $(y, x, x) = 0$ for all $x, y \in R$ and alternative if both condition hold. i.e.$(x, y, y) = 0$ and $(y, y, x) = 0$.
In [2] established that a division ring R is commutative if and only if $[xy, yx] = 0$. Also generalize Guptar's result which assert that a semi prime ring R in which $[xy, yx] = xy^2x - yx^2y \in Z(R)$ or $xy\ o\ xy = xy^2x + yx^2y \in Z(R)$ is necessary commutative, also [1] proved the commutativity of associative ring satisfies the identity $(xy)^2 = xy^2x$. also in their paper proved the properties: $xy\ o\ xy = xy^2x + yx^2y \in Z(R)$ and $xy^nx \pm yx^ny \in Z(R)$,most be commutative In addition, [3] have established the commutativity of non-associative ring satisfying the identities $(xy)^2 = x^2y^2$ and $(xy)^2 \in Z(R) \forall x, y \in R$.
Further, [4] established the commutativity of non-associative primitive rings satisfying the identities: $x(x^2 + y^2)$+$(x^2 + y^2)x \in Z(R)$ and $x(xy)^2 - (xy)^2x \in Z(R)$. Recently [5] show that some results on commutativity of some 2-torsion free non associative rings with unity satisfy:

$(\alpha\beta)^2 - \alpha\beta \in Z(R)\ for\ all\ \alpha\beta\ in\ R$ Motivated by this observation it is natural to look commutativity of alternative rings satisfies
Motivated from them we establish the commutativity of ring with condition (p_1),(p_2) and commutativity of alternative rings with (p_3) with suitable constraint.
Main Result.
The following are the main Results.

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Theorem 2.1.
Suppose that a n-torsion free an alternative ring R with identity 1 and there exist a positive integer n such that: $[xy^nx \pm yx^ny, x] = 0$ or $[xy^nx \pm yx^ny, y] = 0 \quad \forall\, x, y \in . Then\ R$ is commutative.
Proof.
By hypothesis, we have
$[xy^nx \pm yx^ny, y] = 0$. In this property we consider $[xy^nx + yx^ny, y] = 0$
$[yxy^nx - xy^nxy + y^2x^ny - yx^ny^2] = 0$
$[y, (xy^nx)] + [y(x^ny - yx^n)y] <=> [y, xy^nx] + y[x^n, y]y = 0$
Replace $y = y + 1$ in 2.1 above and applied n -torsion free we obtained
$[x\,{}^nC_1y_1x, y] + [x\,{}^nC_2y_2x, y] + \dots\dots\dots\dots\dots + [x\,{}^nC_ny_nx, y]$
$x[\,{}^nC_1y_1 + {}^nC_2y_2 + \dots\dots\dots\dots\dots + {}^nC_ny_n]x = [x^n, y^2] + yx^n = 0$
We used binomial expansion and by inserting $y = y + 1$ for n-times and using the previous we obtained identity in every stage in above we had
$x[({}^{n-1}C_{n-1}{}^{n-1}C_{n-2} + \dots\dots\dots\dots\dots + {}^nC_n)x, y]$.
This gives,
$n!\,[x^2, y] = 0$ in view of n-torsion free condition, we get on the commutator $[x^2, y] = 0$ for $n > 2$ then R is commutative.
Remark 2.2: The following Corollary is an immediate consequence of our main result if we set $n = 2$.
Corollary 2.3
Let R be a 2-torsion free an alternative ring with identity 1. If R has a property:
$[\,xy^2x \pm yx^2y, x] = 0$ or $[xy^2x \pm yx^2y, y] = 0 \ \forall\, x, y \in .$ then R is commutative ring.
Proof
Since $[xy\ o\ xy, y]$, now we consider $[xy^2x - yx^2y, y] = 0$.
$[xy^2x - yx^2y, y] = 0$
$[yxy^2x - xy^2xy + yx^2y^2 - y^2x^2y] = 0$
$[y, xy^2x] + y[x^2, y]y = 0$ 2.3
Replace $y = y + 1$ in 2.3 above and applied 2-torsion free we obtained
$y^2x^2 - x^2y^2 = 0 \ <=> [y^2, x^2] = 0$ 2.4
Replace $y = y + 1$ in 2.4 above we had
$yx^2 - x^2y = 0 \ <=> [y, x^2] = 0$ 2.5
Replace $y = y + 1$ in 2.5 above and applied 2-torsion free we obtained
$yx - xy = 0 \ <=> [y, x] = 0$ or $yx = xy$ implies R is commutative.
Theorem 3.1
Let R be a 2,3-torsion free alternative ring with unity satisfy $[x(xy)^2 + (xy)^2x, x] = 0$, Then R is commutative.

Proof
From the hypothesis above in 3.1
$x[x(xy)^2 + (xy)^2x] = [x(xy)^2 + (xy)^2x]x$, for all x, y ∈ R. 3.2
Substitute $x = (1 + x)$ in , apply 2,3 torsions free and use (1) we get
$y^2x = xy^2$, for all x, y ∈ R. 3.3
Substitute $y = (y + 1)$ in (3.3) and Apply 2-torsion
This implies $xy = yx$ and R is commutative.
Since R is a commutative ring and satisfies the identities either $(xx)y = x\,(xy)$ or $y(xx) = (yx)x$, so that R is an alternative ring. Hence an alternative ring R with identity together with commutativity yields $(x, x, y) = 0 = (y, x, x)$, which completes the proof.

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Conflicts of of interest

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