Babylonian Journal of Mathematics
Vol. **2024**, **pp**. 73–77
DOI:https://doi.org/10.58496/BJM/2024/009; ISSN:3006-113X
https://mesopotamian.press/journals/index.php/mathematics

Research Article

Modified Technique to Solve Degeneracy in Linear Programming Problems

Sadia Qureshi*,[1], Asif Ali Shaikh[1], Sania Qureshi[1],

[1] Department of Basic Science and Related Studies, Mehran University of Engineering and Technology, Jamshoro, Pakistan.

ARTICLE INFO

Article History
Received 22 Jun 2024
Revised: 24 Jul 2024
Accepted 20 Aug 2024
Published 15 Sep 2024

Keywords
Simplex method
Degeneracy
Alternative solution
Modified simplex algorithm
optimum solution

ABSTRACT

This paper addresses degeneracy that has been analyzed in the simplex method in Linear programming (LP) issues. The newly changed technique is proposed for choosing particular pivot rows from leaving variables. This method offers greater beneficial and faster consequences in the assessment of the prevailing classical approach. The proposed set of rules is a higher preference to avoid the confusion of taking an arbitrary ratio to pick out leaving variables and for this reason, the proposed method is robust to solve degeneracy in linear programming (LP) problems.

1. INTRODUCTION

Operation research or operational research popularly called (OR) is the superior analytical technique to enhance selection-making. It is a scientific approach to deciding the finest method for a decision problem beneath the restriction of confined assets. OR models enhance selection-making and decrease the hazard of erroneous choices. Operation research techniques are useful in all varieties of change, business, transportation, product scheduling, an alternative to antique equipment, etc. Furthermore, the most distinguished approach of OR is linear programming. Linear programming (LP) also called linear optimization is a set of rules that was advanced by way of George Dantzig and is one of the simplest strategies to carry out optimization it allows us to clear up several complex issues by making a few simplifying assumptions. Linear programming is the handiest approach for describing complicated relationships via linear capabilities and finding the greatest solutions. This technique allows researchers to find the maximum cost-efficient answers with all barriers and constraints. In mathematical optimization, the Dantzig simplex technique or simplex method is an algorithm for fixing linear programming problems. In keeping with a magazine of "Computing in Science and Engineering" within the twentieth century simplex method is one of the pinnacle10 algorithms. Motzkin who proposed this algorithm derived the simplex method concept and suggested the algorithm's name. The simplex technique is a trendy LP set of rules for optimization problems, regarding a function and several constraints expressed as inequalities.

The standard form of the Simplex algorithm:

Minimize $\mathbf{Z} = c_1x_1 + c_2 x_2 + c_3 x_3 + \ldots + c_n x_n$

Subject to conditions

$$a_{11}x_1 + a_{12} x_2 + a_{13} x_3 + \ldots + a_{1n} x_{1n} (\leq, =, \geq) b_1$$
$$a_{12}x_1 + a_{22} x_2 + a_{23} x_3 + \ldots + a_{2n} x_{2n} (\leq, =, \geq) b_2$$
$$a_{m1}x_1 + a_{m2} x_2 + a_{m3} x_3 + \ldots + a_{mn} x_n (\leq, =, \geq) b_n$$

*Corresponding author. Email: it@gmail.com

and $x_k \geq 0$, k= 1, 2, 3, 4,…, n when c_k , b_m and a_{mk} (k = 1, 2, 3,...,n) are constants and x_k are decision variables and m ≤n . Sometimes an arbitrary ratio happens in the solution column in the simplex technique problems. In such cases, the determination of leaving variable grew to be a query in these sorts of instances degeneracy has been determined and the algorithm grew to be pretty slow. According to Dantzig degenerate solutions are feasible if and solely when the constants of the unique right-hand side aspects endure a unique relation to the coefficients of a basic variable. So, the major reason for this research is to develop a technique that offers the most excellent outcomes in the case of degeneracy. [**1**]Henry Wolkowicz (2023) identifies a specific type of degeneracy at a basic feasible solution (BFS) and explores the theoretical and computational implications of losing strict feasibility in linear programming (LP). They showed numerically that if strict feasibility fails, every BFS becomes degenerate in LP, and the optimality condition remains valid for both the Simplex and interior point methods. Henry Wolkowicz (2022) [2], describes strict feasibility which is assumed for many algorithms for each theory and implementation this writer has used two strategies simplex and interior point methods writer concludes that if strict feasibility fails for a linear program, then each basic feasible solution degenerates. (Masahiro Inuigch 2021) [3], has investigated an approach for computing the vital optimality stages of a non-degenerated basic solution of a linear programming hassle with fuzzy objective function coefficients. In this paper, the researcher discovered that basic feasible solutions were found without any difficulty when the objective function coefficient vector is a non-iterative fuzzy vector.[4]Gerald Gamrath[1,2] (2020) carried out a computational analysis on dual degeneracy within mixed-integer programming (MIP) instances, uncovering its prevalent presence in real-world cases from a standard MIP problem set. The author proposed a new metric, the variable constant ratio, in addition to the proportion of degenerate non-basic variables, to offer a more detailed evaluation of dual degeneracy.(R.SHAIKH 2017) [5] Developed a new approach to unravel the degeneracy in linear programming problems of simplex methods through choosing a precise pivot row whose entries addition is smaller. (Etoa J. 2016) [6], It gave a new pivot rule to clear up a linear programming hassle of the simplex method more effectively than the classical method. It solves the cycling problem in the authentic technique when the problem size is very large. (Nelder and Mead 2015) [7], proposed an algorithm that is about the minimization of a function of variables a manner is used for the estimation of the Hessian matrix. This technique appears to be more effective than the classical method. (Bourab et al.2015) [8], gave a primal algorithm LFP that solves linear applications that lie on linear fractional pricing problems. In the algorithm, dual variables are optimized to locate the largest possible minimum reduced value price at every iteration. (Grover, et al., 2014) [9], discussions have been introduced about the strategies to describe the introduction of interior point methods for students of quite several backgrounds even if they now do not have mathematics majors. (Ping-Qi 2008) [10], an algorithm has been proposed that applies real-world issues of linear programming. In this paper, the writer has introduced an idea for highly degenerated problems in linear programming where bases are not allowed to be in a square matrix this results in too many zero steps solving the real-world problems through the simplex method. (Michael J. 2001) [11] has examined the records of linear programming searching at the techniques such as ellipsoid simplex and different methods, he came to the end that LP problems have a record of reinventing themselves and is very excited to see modification manifest in LP in the subsequent 50 years. (Tomas Gal) [12] developed a technique for degenerate optimum solutions to linear programming problems, the idea of the optimum degeneracy graphs has been developed which explains why industrial courts failed to provide correct outcomes for the cost coefficients and how to overcome it.

2. PROBLEM STATEMENT AND METHODOLOGY

Let Max: $Z= ax_1+bx_2$

Subject to condition: $a_1x_1+ b_1x_2 \leq a\ c_1$

$a_1x_2 + b_2x_2 \leq c_2$

Standard form: using a slack variable to write the equation into standard form

So, Max: $Z- ax_1-bx_2+0S_1+0S_2 = 0$

Subject to: $a_1x_1 + b_1x_2 +S_1 +0S_2 = c_2$

$a_2x_1 + b_2x_2+0S_1+ S_2 = c_2$

	x_1	x_2	S_1	S_2	Solution
Z	-a	-b	0	0	0
S_1	a_1	b_1	0	1	c_1/b_1
S_2	a_2	b_2	1	0	c_2/b_2

X_1 X_2 are non- basic variables
Z, S_1, and S_2 are basic variables
For S_1 entries are: a_1, b_1, 0, 1
For S_2 entries are: a_2, b_2, 1, 0
If –a is the most negative number then we select x_1 as an entering variable,

	X_1	X_2	S_1	S_2	Solution
Z	**-a**	**-b**	**0**	**0**	**0**
S_1	**a_1**	**b_1**	**0**	**1**	**$c_1/b_1=r$**
S_2	**a_2**	**b_2**	**1**	**0**	**$c_2/b_2=r$**

Where r is a constant.
As we know in the simplex method problems smallest ratio is taken from the solution column to choose a leaving variable from the basic variables but in the case where degeneracy occurs, that leads to confusion in selecting a pivot row or column as shown in the table. In such cases, repetition of basic variables may happen. In a cycle of repeated values, the optimal solution lies in the observation that a suitable random choice of pivot row is made if ambiguity occurs. For such a situation, a new modified technique has been proposed to solve degeneracy problems by the simplex method. In this technique, we will select a pivot row whose sum of the coefficients of the objective function must be less than or equal to the coefficients of the basic variables.
Mathematically**:**

(i) If $\sum$ the coefficients of Z $\leq$ $\sum$ of the coefficients of S
(ii) If S_1 has a closer value to the coefficients of Z then we select leaving variable from S_1, otherwise, we select S_2 as a leaving variable
(iii) After selecting a pivot row we repeat the steps of the simplex method until we found the optimum solution.

Examples:

Example (01): An engineering University plains to hire staff members for two departments: computer science and Mathematics. There is total availability of 3 Assistant Professors, 5 lecturers, and 4 lower staff members. The Department of Computer Science requires 3 assistant professors, 5 lecturers, and 1 lower staff member and has 4 lac available money, while the department of Mathematics requires 1 Assistant professor, 3 lecture lower staff members, and has 2 lac available money. Determine how many staff members will the university hire keeping them within the constraints of its resources so that it maximizes the profit.

Resources	**Computer Science**	**Mathematics**	**Availability**
Assistant professors	3	1	3
Lecturers	5	3	5
Lower staff members	1	2	4
Profit/ Available money	400000	200000	

Let x_1 be the no. of staff members in the Computer science department hired by the university & x_2 is no. of staff members in the Mathematics department hired by the University.
Max: $Z = 400000X_1 + 200000X_2$
Subject to constraints:

$3x_1 + x_2 \leq 3$
$5x_1 + 3x_2 \leq 5$
$x_1 + 2x_2 \leq 4$
$x_1, x_2 \geq 0$

Results: Z= 400000, x_1= 1 x_2= 0 (by proposed method)
Status: Verified
Hence 1 staff member has been hired by the Computer science department and no staff has been hired by the Mathematics department keeping within the constraints of its resources so that it maximizes the profit.

Example (02): Max: $Z = 3x_1 + 9x_2$
Subject to constraints: $x_1 + 4x_2 \leq 8$

$x_1 + 2x_2 \leq 4$
$x_1, x_2 \geq 0$

Results: Z = 18, $x_1 = 0$, $x_2 = 2$
Status: Verified

Example (03) Max $Z = 2x_1 + x_2$
Subject to constraints: $3x_1 + x_2 \leq 3$
$4x_1 + 3x_2 \leq 6$
$3x_1 + 2x_2 \leq 3$
$x_1, x_2 \geq 0$

Results: Z = 2, $x_1 = 1$, $x_2 = 0$
Status: Verified

3. RESULTS AND DISCUSSION

Examples	Simplex Method	Modified algorithm	Discussion
Max $Z= 400000x_1 + 200000x_2$ Subject to: $3x_1 + x_2 \leq 3$ $5x_1+ 3x_2 \leq 5$ $x_1 + 2x_2 \leq 4$ $x_1 , x_2 \geq 0$	Results: Z= 400000 $x_1 = 1$, $x_2 = 0$ Status: Verified	Results: Z= 400000 $x_1 = 1$, $x_2 = 0$ Status: Verified	In the modified algorithm, the confusion of selecting an arbitrary value has been removed by selecting a fixed value from a pivot row and we can also get all the required results by using the process of the proposed method.
Max $Z = 3x_1 + 9x_2$ Subject to: $x_1 + 4x_2 \leq 8$ $x_1 + 2x_2 \leq 4$ $x_1 , x_2 \geq 0$	Results: Z = 18 $x_1= 0$, $x_2 = 2$ Status: Verified	Results: Z= 18 $x_1= 0$, $x_2 = 2$ Status: Verified	
Max $Z = 2x_1+ x_2$ Subject to: $3x_1 + x_2 \leq 3$ $4x_1 + 3x_2 \leq 6$ $3x_1 + 2x_2 \leq 3$ $x_1 , x_2 \geq 0$	Results: Z = 2 $x_1 = 1$, $x_2 = 0$ Status: Verified	Results: Z= 2 $x_1 = 1$, $x_2 = 0$ Status: Verified	

4. CONCLUSION

The proposed research is limited to the exact solutions of degenerate maximization of linear programming problems using the simplex method. A challenge arises with degeneracy in the simplex method, as it complicates the selection of a set of leaving variables. To address this issue, a new technique has been introduced that selects a fixed leaving variable to achieve the desired results. Consequently, this new technique is used to determine the values of all non-basic variables, which also saves our time compared to the existing method that considers the missing variable as zero.

Conlicts Of Interest

The authors declare no conflicts of interest.

Funding

This research received no external funding.

Acknowledgment

The authors express heartfelt appreciation to the institution for their unwavering moral support and encouragement throughout this research endeavor.

References

[1] H. Wolkowicz , and Haesol, "Revisiting Degeneracy, Strict Feasibility, Stability, in Linear Programming.

[2] Henry Wolkowicz, Jiyoung Im* + arXiv 2203.02795V1[math OC](2022) Strict feasibility and degeneracy in a linear programming problem.

[3] Robust optimality analysis of non-degenerate basic feasible solutions in linear programming problems with fuzzy objectives coefficients. Masahiro Inuigch[1] Zhenzhong Goa[1].Carla Olivier henenriques[234] accepted 27 December (2021).

[4] Gerald Gamrath[1,2]. Timo Berthold[3.] Domenico Salvagin[4] (2020) An exploratory computational analysis of dual degeneracy in mixed-integer programming

[5] R. SHAIKH (2017) Development of a new technique to solve degeneracy in Linear programming problems Sindh Univ. Res. Jour. (Sci. Ser.) Vol. 49(3) 571-574

[6] Etoa J. (2016) New Optimal Pivot Rule for the Simplex Algorithm, University of Yaounde 2, Soa, Cameroon, 6, 647-658.

[7] Nelder J A and R Mead (2015) A Simplex method for function minimization, Churchill College Cambridge 545-554.

[8] Hocine B., G. Desaulniers and J. Desrosiers (2015) A linear fractional pricing problem for solving linear programs, Polytechnique and HEC Montreal and GERAD, Canada.

[9] Grover, R., N. Kumar, and V. Saini (2014) Linear programming, Dronachaya College of Engineering, Gurgaon, IJIRT, 2349-6002.

[10] Ping-Qi P (2008) A primal deficient- basis simplex algorithm for linear programming Southeast University, Nanjing 210096, People's Republic of China.

[11] J. Michael, "The many facets of Linear Programming", School of OR &IE, Cornell University, Ithaca, NY, 2021, 14853-3801, USA

[12] T. Gal, "Degeneracy Problems in Mathematical Programming and Degeneracy graphs", Department of Quantitative Management South Africa, Pretoria ORION, Vol. 6 No. 1, pp.3-36, December 2003.