



Research Article

Results for $\mathcal{SL}(2, p)$, $p = 3, 5$ and 7

Niran Sabah Jasim^{1,*}, Mohammed Serdar I. Kirdar², Mohammed Yasin³, Shrooq Bahjat Smecin⁴, Amal Rashid Al Hajri⁵

¹ Department of Mathematics, College of Education for Pure Science Ibn Al-Haitham, University of Baghdad, Baghdad, Iraq.

² Applied Science Department, University of Technology, Baghdad, Iraq.

³ Department of Mathematics, An-Najah National University, Nablus P400, Palestine.

⁴ Information Department -Section Mathematics, University of Technology and Applied science - Muscat, Sultanate of Oman.

⁵ Department of Information Technology/Mathematics Section, University of Technology and Applied Sciences- Muscat, Oman.

ARTICLE INFO

Article History

Received: 07 Sep 2024

Revised: 05 Oct 2024

Accepted: 05 Nov 2024

Published: 30 Nov 2024

Keywords

restricted group

representation model

cyclic decomposition



ABSTRACT

The collection of whole $\mathcal{Z}$ - account grade maps respecting a restricted group G of commutative group $\text{cf}(G, \mathcal{Z})$ beneath spot wise addendum. Into these one group own $\mathcal{Z}$ - account generalized characters a subgroup indicate $R(G)$.

Whole invertible $n \times n$ model form a group on a field F indicate $\mathcal{GL}(n, F)$. A homo. of $\mathcal{GL}(n, F)$ to $F - \{0\}$ is the determinant of these model, $\mathcal{SL}(n, F)$ indicate the kernel of it. Thus (n, F) is a subgp. of $\mathcal{GL}(n, F)$ include whole models of determinant 1 on F .

Let V vector void on F , $\mathcal{GL}(V)$ indicate whole linear isomorphism of V onto itself, a representation of G for representation void V is a homo. of G to $\mathcal{GL}(V)$. A representation model of G is a homo. of G to $\mathcal{GL}(n, F)$, where n is the degree of the representation model.

From the rational representations character table (CTRR) we compute the cyclic decomposition (CD) for $p = 3, 5$, and 7 to $\mathcal{SL}(2, p)$.

1. INTRODUCTION

The significance of character and representation notion for survey of group's proceeds on one duke to the reality ought to be indispensable to offer a fixed depiction of a group; it can accomplish together with a model representation. Another duke, group notion profit at most to the employ of characters representations and, while this oncoming are utilize as a further to resolve the constructing a group. Furthermore, character and representation notion supply diverse applications, not exclusive in other offshoot of mathematics but as well in chemistry and physics. [1].

This work found the (CD) for $\mathcal{SL}(2, p)$, where $p = 3, 5$, and 7 after we find the (CTRR) for each group and diagonal the matrix of this (CTRR) if we suppose that the terms of this basic diagonal are $a, b, c, \dots, n$ then the (CD) is $\mathcal{Z}_a \oplus \mathcal{Z}_b \oplus \mathcal{Z}_c \oplus \dots \oplus \mathcal{Z}_n$.

2. BACKGROUND

Theorem 2.1: [2]

the group has order $p^k (p^{2k} - 1)$.

Theorem 2.2: [2]

The conjugacy classes is satisfied by the following table:

*Corresponding author. Email: niraan.s.j@ihcoedu.uobaghdad.edu.iq

TABLE I. THE CONJUGACY CLASSES IS SATIFIED

$g \in G$	Notation	C_g	$ C_g $	$ C_G(g) $
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	1	C_1	1	$p^k(p^{2k}-1)$
$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	z	C_z	1	$p^k(p^{2k}-1)$
$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$	c	C_c	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} 1 & 0 \\ \nu & 1 \end{pmatrix}$	d	C_d	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}$	zc	C_{zc}	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} -1 & 0 \\ -\nu & -1 \end{pmatrix}$	zd	C_{zd}	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} \nu^\ell & 0 \\ 0 & \nu^{-\ell} \end{pmatrix}$	a^ℓ	Ca^ℓ	$p^k(p^k+1)$	p^k-1
Element of order $(p^k+1)/m$	b^m	Cb^m	$p^k(p^k-1)$	p^k+1

From [3]

$$\mu(n) = \begin{cases} 1 & \text{if } n=1 \\ 0 & \text{if } a^2/n \text{ for some } a>1 \\ (-1)^K & \text{if } n = p_1 p_2 \cdots p_K, p_i \text{ are distinct primes.} \end{cases}$$

Lemma (2.3): [1].

For $G = \mathcal{SL}(2, p^k)$, $e, e' < (p^k - 1)/2$ and $f, f' < (p^k + 1)/2$, also $\varepsilon = (-1)^{(p^k-1)/2}$, let $\rho, \sigma \in \mathbb{C}$ are $(p^k - 1)$ and $(p^k + 1)$ -th origin of 1 resp.

$$\mathcal{B}(\ell) = \begin{cases} 1 & \text{if } \ell \text{ is even} \\ 2 & \text{otherwise} \end{cases}, \quad E(p^k) = \begin{cases} 1 & \text{if } p^k \equiv 3 \pmod{4} \\ 2 & \text{otherwise} \end{cases}, \quad \mathcal{A}(e) = (1/2)\mathcal{O}((p^k - 1)/e),$$

$$\mathcal{C}(f) = (1/2)\mathcal{O}((p^k + 1)/f), \quad \tau_1(e, e') = [\mathcal{O}((p^k - 1)/e)/\mathcal{O}((p^k - 1)/e e')] \mu((p^k - 1)/e e'), \quad \tau_2(f, f') = [\mathcal{O}((p^k + 1)/f)/\mathcal{O}((p^k + 1)/f f')] \mu((p^k + 1)/f f')$$

The (CTRR) is

TABLE II. THE CTRR.

Cg	1	z	c and d	$a^{e'}$	$b^{f'}$
$ Cg $	1	1	$(p^{2k}-1)/2$	$p^k(p^k+1)$	$p^k(p^k-1)$
$ CG(g) $	$p^k(p^{2k}-1)$	$p^k(p^{2k}-1)$	$2p^k$	p^k-1	p^k+1
1_G	1	1	1	1	1
ψ	p^k	p^k	0	1	-1
γ_e	$(p^k+1)A(e)B(e)$	$(-1)^e(p^k+1)A(e)B(e)$	$A(e)B(e)$	$B(e)\tau_1(e,e')$	0
θ_f	$(p^k-1)C(f)B(f)$	$(-1)^f(p^k-1)C(f)B(f)$	$-C(f)B(f)$	0	$-B(f)\tau_2(f,f')$
$\xi_1 + \xi_2$	(p^k+1)	$\varepsilon(p^k+1)$	1	$(-1)^{e'}2$	0
$\eta_1 + \eta_1$	$(p^k-1)E(p^k)$	$-\varepsilon(p^k-1)E(p^k)$	-1	0	$(-1)^{f'+1}2E(p^k)$

Theorem (2.4): [4,5]

$$K(G) = \bigoplus_{i=1}^n \mathbb{Z} P^i.$$

3. THE RESULTS

3.1 The (CD) of $\mathcal{SL}(2,3)$

The (CTRR) is :

C_g	1	z	c	zc	b
$ C_g $	1	1	4	4	6
$ CG(g) $	24	24	6	6	4
1_G	1	1	1	1	1
ψ	3	3	0	0	-1
θ_1	2	-2	-1	1	0
$\xi_1 + \xi_2$	4	-4	1	-1	0
$\eta_1 + \eta_2$	2	2	-1	-1	2

Then the diagonal of it is

$$\begin{pmatrix} 24 & 0 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

Thus by (*), we obtained

$$K(\mathcal{SL}(2,3)) = \mathbb{Z}_{24} \oplus \mathbb{Z}_6 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_1 \oplus \mathbb{Z}_1$$

3.2 The (CD) of $\mathcal{SL}(2,5)$

The (CTRR) is :

C_g	1	z	c	zc	a	b	b^2
$ C_g $	1	1	12	12	30	20	20
$ C_G(g) $	120	120	10	10	4	6	6
1_G	1	1	1	1	1	1	1
ψ	5	5	0	0	1	-1	-1
χ	6	-6	1	-1	0	0	0
θ_1	4	-4	-1	1	0	-1	1
θ_2	4	4	-1	-1	0	1	1
ξ'	6	6	1	1	-2	0	0
η'	4	-4	-1	1	0	2	-2

Then the diagonal of it is

$$\begin{pmatrix} 120 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -30 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

Thus by (*), we obtained

$$K(\mathcal{SL}(2,5)) = Z_{120} \oplus Z_{30} \oplus Z_2 \oplus Z_1 \oplus Z_1 \oplus Z_1 \oplus Z_2$$

3.3 The (CD) of $\mathcal{SL}(2,7)$

The (CTRR) is :

C_g	1	z	c	zc	a	a^2	b	b^2
$ C_g $	1	1	24	24	56	56	42	42
$ CG(g) $	336	336	14	14	6	6	8	8
1_G	1	1	1	1	1	1	1	1
ψ	7	7	0	0	1	1	-1	-1
χ_1	8	-8	1	-1	1	-1	0	0
χ_2	8	8	1	1	-1	-1	0	0
θ_1	12	-12	-2	2	0	0	0	0
θ_2	6	6	-1	-1	0	0	0	2
$\xi_1 + \xi_2$	8	-8	1	-1	-2	2	0	0
$\eta_1 + \eta_2$	6	6	-1	-1	0	0	2	-2

Then the diagonal of it is

$$\begin{pmatrix} 336 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -84 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 \end{pmatrix}$$

Thus by (*), we obtained

$$K(\mathcal{SL}(2,7)) = Z_{336} \oplus Z_{84} \oplus Z_2 \oplus Z_1 \oplus Z_1 \oplus Z_2 \oplus Z_1 \oplus Z_2$$

Conflicts Of Interest

There are no conflicts of interest.

Funding

There is no funding for the paper.

Acknowledgment

The authors extend their sincere thanks to the editor and members of the preparatory committee of the Babylonian Journal of mathematics.

References

- [1] J.P.Serre; “Linear Representation Of Finite Groups”, Springer-Verlage, 1977.
- [2] K.E.Gehles, “Ordinary Characters of Finite Special Linear Groups”, M.Sc. Dissertation, University of ST. Andrews; 2002.
- [3] M.S.Kirdar; “On Brauer’s Proof Of The Artin Induction Theorem”, Abhath AL–Yarmouk (Basic Sciences and Engineering), Yarmouk University, Vol.11, No.1A, pp.51–54, 2002.
- [4] H.Behravesh, “The Rational Character Table Of Special Linear Groups”, J.Sci.I.R.Iran, Vol.9, No.2, pp.173 – 180; 1998.
- [5] M.S.Kirdar, “The Factor Group of the Z-Valued Class Function Module The Group of the Generalized Characters”, Ph.D. Thesis, University of Birmingham; 1982.