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Research Article

Consequence for the Partition (9,9,3)

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ABSTRACT

Let $\mathcal{F}$ be a free module defined on the commutative ring $\mathcal{R}$ with identity. Buchsbaum studied the Weyl module resolution where the Weyl module $\mathcal{K}_{\lambda/\mu}(\mathcal{F})$ is the image of the Weyl map $d'_{\lambda/\mu}(\mathcal{F})$ for the skew-partition λ/μ ; where λ runs over all partitions $\lambda=(\lambda_1,\lambda_2,\dots,\lambda_s)$. There are a number of classical formulas that express the formal character of the representation $\mathcal{L}_\lambda(\mathcal{F})$ in terms of standard symmetric polynomials. Such formulas are also valid for the more general representation modules $\{\mathcal{L}_{\lambda/\mu}(\mathcal{F})\}$ associated to skew partition λ/μ; where $\mu\subseteq\lambda$, where the set of all irreducible polynomial representations of general linear group $\mathrm{GL}_n(\mathcal{F})$ of degree n is described by the module $\{\mathcal{L}_\lambda(\mathcal{F})\}$
For the partition (9,9,3) by applying the boundary maps we reduction the terms of the characteristic-free of Weyl module resolution to the terms of the Lascoux resolution and prove that the sequence of the reduction terms is exact.

1. INTRODUCTION

The author in [1] described the resolutions $\widetilde{\mathcal{B}}_{\bullet}$ of Weyl modules by writing down explicit projective resolutions of the two-rowed modules. The existence proof of resolution for the similar problem with an arbitrary number of rows the researchers in [2] gave that. While the existence of resolution of Weyl module whose terms are direct sums of tensor products of divided powers proved by the authors in [3].
Hassan generalized the techniques in [4] for the partitions (3,3,3), and (4,4,3) in [5,6] respectively, also authors in [7-9] studied the cases (8,7,3), (6,6,4;0,0), (7,7,4;0,0).
For the partition (9,9,3) the terms of Weyl module from characteristic-free resolution reduction to the terms of Lascoux resolution and prove that the sequence of these terms is exact in this work.

2. RESOLUTION OF THE CHARACTERISTIC-FREE AND LASCOUX

The terms of the resolution for the partition (9,9,3), [4].

$$Res([9,9;0])\otimes\mathcal{D}_3\oplus\sum_{e\geq0}\mathcal{Z}_{32}^{(e+1)}y\,Res([9,9+e+1;e+1])\otimes\mathcal{D}_{3-e-1}\oplus$$
$$\sum_{e_1\geq0,e_2\geq e_1}\mathcal{Z}_{32}^{(e_2+1)}y\mathcal{Z}_{31}^{(e_1+1)}z Res([9+e_1+1,9+e_2+1;e_2-e_1])\otimes\mathcal{D}_{3-(e_1+e_2+2)} \quad (1)$$

So

$$\sum_{e\geq0}\mathcal{Z}_{32}^{(e+1)}y\,Res([9,9+e+1;e+1])\otimes\mathcal{D}_{3-e-1}=\mathcal{Z}_{32}y\,Res([9,10;1])\otimes\mathcal{D}_2\oplus$$

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$\underline{\mathcal{Z}}_{32}^{(2)}\mathcal{y}\ Res([9,11;2])\otimes\mathcal{D}_1\oplus\underline{\mathcal{Z}}_{32}^{(3)}\mathcal{y}\ Res([9,12;3])\otimes\mathcal{D}_0$,
and
$\sum_{e_1\geq 0, e_2\geq e_1}\ \underline{\mathcal{Z}}_{32}^{(e_2+1)}\mathcal{y}\underline{\mathcal{Z}}_{31}^{(e_1+1)}\mathcal{z}Res([9+e_1+1, 9+e_2+1; e_2-e_1])\otimes$
$\mathcal{D}_{3-(e_1+e_2+2)} = \underline{\mathcal{Z}}_{32}\mathcal{y}\underline{\mathcal{Z}}_{31}\mathcal{z}Res([10,10;0])\otimes\mathcal{D}_1\oplus\underline{\mathcal{Z}}_{32}^{(2)}\mathcal{y}\underline{\mathcal{Z}}_{31}\mathcal{z}Res([10,11;1])\otimes\mathcal{D}_0$;
where $\underline{\mathcal{Z}}_{32}\mathcal{y}$ is the Bar complex:

$$0 \longrightarrow \mathcal{Z}_{32}\mathcal{y} \xrightarrow{\partial_{\mathcal{y}}} \mathcal{Z}_{32} \longrightarrow 0,$$

$\underline{\mathcal{Z}}_{32}^{(2)}\mathcal{y}$ is the Bar complex:

$$0 \longrightarrow \mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y} \xrightarrow{\partial_{\mathcal{y}}} \mathcal{Z}_{32}^{(2)}\mathcal{y} \xrightarrow{\partial_{\mathcal{y}}} \mathcal{Z}_{32}^{(2)} \longrightarrow 0,$$

$\underline{\mathcal{Z}}_{32}^{(3)}\mathcal{y}$ is the Bar complex:

$$0 \longrightarrow \mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y} \xrightarrow{\partial_{\mathcal{y}}} \begin{matrix} \mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{32}\mathcal{y} \\ \oplus \\ \mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}^{(2)}\mathcal{y} \end{matrix} \xrightarrow{\partial_{\mathcal{y}}} \mathcal{Z}_{32}^{(3)}\mathcal{y} \xrightarrow{\partial_{\mathcal{y}}} \mathcal{Z}_{32}^{(3)} \longrightarrow 0,$$

and $\underline{\mathcal{Z}}_{31}\mathcal{z}$ is the Bar complex:

$$0 \longrightarrow \mathcal{Z}_{31}\mathcal{z} \xrightarrow{\partial_{\mathcal{z}}} \mathcal{Z}_{31} \longrightarrow 0;$$

where $\mathcal{x}$, $\mathcal{y}$ and $\mathcal{z}$ stand for the separator variables, and the boundary map is $\partial_{\mathcal{x}}+\partial_{\mathcal{y}}+\partial_{\mathcal{z}}$.
Let Bar $(\mathcal{M}, \mathcal{A}; \mathcal{S})$ be the free Bar module on the set $\mathcal{S}=\{\mathcal{x}, \mathcal{y}, \mathcal{z}\}$; where $\mathcal{A}$ is the free associative algebra generated by $\mathcal{Z}_{21}$, $\mathcal{Z}_{32}$, and $\mathcal{Z}_{31}$ and their divided powers with the following relations:

$$\mathcal{Z}_{32}^{(a)}\mathcal{Z}_{31}^{(b)} = \mathcal{Z}_{31}^{(b)}\mathcal{Z}_{32}^{(a)} \quad \text{and} \quad \mathcal{Z}_{21}^{(a)}\mathcal{Z}_{31}^{(b)} = \mathcal{Z}_{31}^{(b)}\mathcal{Z}_{21}^{(a)}.$$

And the module $\mathcal{M}$ is the direct sum of $\mathcal{D}_{\mathcal{p}}\otimes\mathcal{D}_{\mathcal{q}}\otimes\mathcal{D}_{\mathcal{r}}$ for suitable $\mathcal{p}, \mathcal{q}$, and $\mathcal{r}$ with the action of $\mathcal{Z}_{21}$, $\mathcal{Z}_{32}$, and $\mathcal{Z}_{31}$ and their divided powers.
The terms of the characteristic-free resolution (4.3.1); where $b, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, c_1, c_2 \in \mathbb{Z}^+$ are:

∘ In dimension zero $(\mathcal{X}_0)$ we have $\mathcal{D}_9\otimes\mathcal{D}_9\otimes\mathcal{D}_3$.

∘ In dimension one $(\mathcal{X}_1)$ we have the sum of the following terms:
- $\mathcal{Z}_{21}^{(b)}\mathcal{x}\mathcal{D}_{9+b}\otimes\mathcal{D}_{9-b}\otimes\mathcal{D}_3$; where $1\leq b\leq 9$.
- $\mathcal{Z}_{32}^{(b)}\mathcal{y}\mathcal{D}_9\otimes\mathcal{D}_{9+b}\otimes\mathcal{D}_{3-b}$; where $1\leq b\leq 3$.

∘ In dimension two $(\mathcal{X}_2)$ we have the sum of the following terms:
- $\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_3$; where $2\leq|b|=b_1+b_2\leq 9$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b)}\mathcal{x}\mathcal{D}_{9+b}\otimes\mathcal{D}_{10-b}\otimes\mathcal{D}_2$; where $2\leq b\leq 10$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(b)}\mathcal{x}\mathcal{D}_{9+b}\otimes\mathcal{D}_{11-b}\otimes\mathcal{D}_1$; where $3\leq b\leq 11$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(b)}\mathcal{x}\mathcal{D}_{9+b}\otimes\mathcal{D}_{12-b}\otimes\mathcal{D}_0$; where $4\leq b\leq 12$.
- $\mathcal{Z}_{32}^{(b_1)}\mathcal{y}\mathcal{Z}_{32}^{(b_2)}\mathcal{y}\mathcal{D}_9\otimes\mathcal{D}_{12+|b|}\otimes\mathcal{D}_{3-|b|}$; where $2\leq|b|=b_1+b_2\leq 3$.
- $\mathcal{Z}_{32}^{(b)}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{D}_{10}\otimes\mathcal{D}_{9+b}\otimes\mathcal{D}_{2-b}$; where $1\leq b\leq 2$.

∘ In dimension three $(\mathcal{X}_3)$ we have the sum of the following terms:
- $\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_3$; where $3\leq|b|=\sum_{i=1}^{3}b_i\leq 9$ and $b_1\geq 1$.
- $\mathcal{Z}_{32}\mathcal{y}\ \mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_2$; where $3\leq|b|=b_1+b_2\leq 10$ and $b_1\geq 2$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_1$; where $4\leq|b|=b_1+b_2\leq 11$ and $b_1\geq 3$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b)}\mathcal{x}\mathcal{D}_{9+b}\otimes\mathcal{D}_{11-b}\otimes\mathcal{D}_1$; where $3\leq b\leq 11$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_0$; where $5\leq|b|=b_1+b_2\leq 12$ and $b_1\geq 4$.
- $\mathcal{Z}_{32}^{(c_1)}\mathcal{y}\mathcal{Z}_{32}^{(c_2)}\mathcal{y}\mathcal{Z}_{21}^{(b)}\mathcal{x}\mathcal{D}_{9+b}\otimes\mathcal{D}_{12-b}\otimes\mathcal{D}_0$; where $c_1+c_2=3$ and $4\leq b\leq 12$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{D}_9\otimes\mathcal{D}_{12}\otimes\mathcal{D}_0$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b)}\mathcal{x}\mathcal{D}_{10+b}\otimes\mathcal{D}_{10-b}\otimes\mathcal{D}_1$; where $1\leq b\leq 10$.

- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b)}\mathcal{x}\mathcal{D}_{10+b}\otimes\mathcal{D}_{11-b}\otimes\mathcal{D}_{0}$; where $2 \leq b \leq 11$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{D}_{10}\otimes\mathcal{D}_{11}\otimes\mathcal{D}_{0}$.

◦ In dimension four $(\mathcal{X}_4)$ we have the sum of the following terms:

- $\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_{3}$; where $4 \leq |b| = \sum_{i=1}^{4} b_i \leq 9$ and $b_1 \geq 1$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_{2}$; where $4 \leq |b| = \sum_{i=1}^{3} b_i \leq 10$ and $b_1 \geq 2$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_{1}$; where $5 \leq |b| = \sum_{i=1}^{3} b_i \leq 11$ and $b_1 \geq 3$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_{1}$; where $4 \leq |b| = b_1 + b_2 \leq 11$; and $b_1 \geq 3$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_{0}$; where $6 \leq |b| = \sum_{i=1}^{3} b_i \leq 12$ and $b_1 \geq 4$.
- $\mathcal{Z}_{32}^{(c_1)}\mathcal{y}\mathcal{Z}_{32}^{(c_2)}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_{0}$; where $c_1 + c_2 = 3$, $5 \leq |b| = b_1 + b_2 \leq 12$ and $b_1 \geq 4$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b)}\mathcal{x}\mathcal{D}_{9+b}\otimes\mathcal{D}_{12-b}\otimes\mathcal{D}_{0}$; where $4 \leq b \leq 12$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_{1}$; where $2 \leq |b| = b_1 + b_2 \leq 10$ and $b_1 \geq 1$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_{0}$; where $3 \leq |b| = b_1 + b_2 \leq 11$ and $b_1 \geq 2$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b)}\mathcal{x}\mathcal{D}_{10+b}\otimes\mathcal{D}_{11-b}\otimes\mathcal{D}_{0}$; where $2 \leq b \leq 11$.

◦ In dimension five $(\mathcal{X}_5)$ we have the sum of the following terms:

- $\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_{3}$; where $5 \leq |b| = \sum_{i=1}^{5} b_i \leq 9$ and $b_1 \geq 1$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_{2}$; where $5 \leq |b| = \sum_{i=1}^{4} b_i \leq 10$ and $b_1 \geq 2$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_{1}$; where $6 \leq |b| = \sum_{i=1}^{4} b_i \leq 11$ and $b_1 \geq 3$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_{1}$; where $5 \leq |b| = \sum_{i=1}^{3} b_i \leq 11$ and $b_1 \geq 3$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_{0}$; where $7 \leq |b| = \sum_{i=1}^{4} b_i \leq 12$ and $b_1 \geq 4$.
- $\mathcal{Z}_{32}^{(c_1)}\mathcal{y}\mathcal{Z}_{32}^{(c_2)}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_{0}$; where $c_1 + c_2 = 3$, $6 \leq |b| = \sum_{i=1}^{3} b_i \leq 12$ and $b_1 \geq 4$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_{0}$; where $5 \leq |b| = b_1 + b_2 \leq 12$ and $b_1 \geq 4$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_{1}$; where $3 \leq |b| = \sum_{i=1}^{3} b_i \leq 10$ and $b_1 \geq 1$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_{0}$; where $4 \leq |b| = \sum_{i=1}^{3} b_i \leq 11$ and $b_1 \geq 2$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_{0}$; where $3 \leq |b| = b_1 + b_2 \leq 11$ and $b_1 \geq 2$.

◦ In dimension six $(\mathcal{X}_6)$ we have the sum of the following terms:

- $\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_{3}$; where $6 \leq |b| = \sum_{i=1}^{6} b_i \leq 9$ and $b_1 \geq 1$.
- $\mathcal{Z}_{32}\mathcal{y}\ \mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_{2}$; where $6 \leq |b| = \sum_{i=1}^{5} b_i \leq 10$ and $b_1 \geq 2$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\ \mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_{1}$; where $7 \leq |b| = \sum_{i=1}^{5} b_i \leq 11$ and $b_1 \geq 3$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_{1}$; where $6 \leq |b| = \sum_{i=1}^{4} b_i \leq 11$ and $b_1 \geq 3$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\ \mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_{0}$; where $8 \leq |b| = \sum_{i=1}^{5} b_i \leq 12$ and $b_1 \geq 4$.
- $\mathcal{Z}_{32}^{(c_1)}\mathcal{y}\mathcal{Z}_{32}^{(c_2)}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_{0}$; where $c_1 + c_2 = 3$, $7 \leq |b| = \sum_{i=1}^{4} b_i \leq 12$ and $b_1 \geq 4$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_{0}$; where $6 \leq |b| = \sum_{i=1}^{3} b_i \leq 12$ and $b_1 \geq 4$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_{1}$; where $4 \leq |b| = \sum_{i=1}^{4} b_i \leq 10$ and $b_1 \geq 1$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_{0}$; where $5 \leq |b| = \sum_{i=1}^{4} b_i \leq 11$ and $b_1 \geq 2$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_{0}$; where $4 \leq |b| = \sum_{i=1}^{3} b_i \leq 11$ and $b_1 \geq 2$.

◦ In dimension seven $(\mathcal{X}_7)$ we have the sum of the following terms:

- $\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_{3}$; where $7 \leq |b| = \sum_{i=1}^{7} b_i \leq 9$ and $b_1 \geq 1$.
- $\mathcal{Z}_{32}\mathcal{y}\ \mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_{2}$; where $7 \leq |b| = \sum_{i=1}^{6} b_i \leq 10$ and $b_1 \geq 2$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\ \mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_{1}$; where $8 \leq |b| = \sum_{i=1}^{6} b_i \leq 11$ and $b_1 \geq 3$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_{1}$; where $7 \leq |b| = \sum_{i=1}^{5} b_i \leq 11$ and $b_1 \geq 3$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\ \mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_{0}$; where $9 \leq |b| = \sum_{i=1}^{6} b_i \leq 12$ and $b_1 \geq 4$.

- $\mathcal{Z}_{32}^{(c_1)}\mathcal{y}\mathcal{Z}_{32}^{(c_2)}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_0$; where $c_1+c_2=3$, $8\le|b|=\sum_{i=1}^{5}b_i\le 12$ and $b_1\ge 4$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_0$; where $7\le|b|=\sum_{i=1}^{4}b_i\le 12$ and $b_1\ge 4$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_1$; where $5\le|b|=\sum_{i=1}^{5}b_i\le 10$ and $b_1\ge 1$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $6\le |b|=\sum_{i=1}^{5}b_i\le 11$ and $b_1\ge 2$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $5\le|b|=\sum_{i=1}^{4}b_i\le 11$ and $b_1\ge 2$.

◦ In dimension eight $(\mathcal{X}_8)$ we have the sum of the following terms:

- $\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{Z}_{21}^{(b_8)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_3$; where $8\le|b|=\sum_{i=1}^{8}b_i\le 9$ and $b_1\ge 1$.
- $\mathcal{Z}_{32}\mathcal{y}\ \mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_2$; where $8\le|b|=\sum_{i=1}^{7}b_i\le 10$ and $b_1\ge 2$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\ \mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_1$; where $9\le|b|=\sum_{i=1}^{7}b_i\le 11$ and $b_1\ge 3$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_1$; where $8\le|b|=\sum_{i=1}^{6}b_i\le 11$ and $b_1\ge 3$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\ \mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_0$;where $10\le|b|=\sum_{i=1}^{7}b_i\le 12$and $b_1\ge 4$.
- $\mathcal{Z}_{32}^{(c_1)}\mathcal{y}\mathcal{Z}_{32}^{(c_2)}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_0$; where $c_1+c_2=3$, $9\le|b|=\sum_{i=1}^{6}b_i\le 12$ and $b_1\ge 4$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_0$; where $8\le|b|=\sum_{i=1}^{5}b_i\le 12$ and $b_1\ge 4$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_1$; where $6\le|b|=\sum_{i=1}^{6}b_i\le 10$ and $b_1\ge 1$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $7\le |b|=\sum_{i=1}^{6}b_i\le 11$ and $b_1\ge 2$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $6\le|b|=\sum_{i=1}^{5}b_i\le 11$ and $b_1\ge 2$.

◦ In dimension nine $(\mathcal{X}_9)$ we have the sum of the following terms:

- $\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{D}_{18}\otimes\mathcal{D}_0\otimes\mathcal{D}_0$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{Z}_{21}^{(b_8)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_2$;
  where $9\le|b|=\sum_{i=1}^{8}b_i\le 10$ and $b_1\ge 2$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{Z}_{21}^{(b_8)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_1$;
  where $10\le|b|=\sum_{i=1}^{8}b_i\le 11$ and $b_1\ge 3$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_1$;
  where $9\le|b|=\sum_{i=1}^{7}b_i\le 11$ and $b_1\ge 3$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{Z}_{21}^{(b_8)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_0$;
  where $11\le |b|=\sum_{i=1}^{8}b_i\le 12$ and $b_1\ge 4$.
- $\mathcal{Z}_{32}^{(c_1)}\mathcal{y}\mathcal{Z}_{32}^{(c_2)}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_0$;
  where $c_1+c_2=3$, $11\le|b|=\sum_{i=1}^{7}b_i\le 12$ and $b_1\ge 4$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_0$;where $9\le|b|\le 12$ and $b_1\ge 4$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_1$;
  where $7\le |b|=\sum_{i=1}^{7}b_i\le 10$ and $b_1\ge 1$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$;
  where $8\le|b|=\sum_{i=1}^{3}b_i\le 11$ and $b_1\ge 2$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$;where $7\le|b|\le 11$ and $b_1\ge 2$.

◦ In dimension ten $(\mathcal{X}_{10})$ we have the sum of the following terms:

- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{D}_{19}\otimes\mathcal{D}_0\otimes\mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}\mathcal{x}\mathcal{Z}_{21}^{(3)}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{D}_{20}\otimes\mathcal{D}_0\otimes\mathcal{D}_1$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{Z}_{21}^{(b_8)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_1$;
  where $10\le|b|=\sum_{i=1}^{8}b_i\le 11$ and $b_1\ge 3$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\ \mathcal{Z}_{21}^{(4)}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{D}_{21}\otimes\mathcal{D}_0\otimes\mathcal{D}_0$
- $\mathcal{Z}_{32}^{(c_1)}\mathcal{y}\mathcal{Z}_{32}^{(c_2)}\mathcal{y}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{Z}_{21}^{(b_8)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_0$;

where $c_1 + c_2 = 3$, $11 \le |b| = \sum_{i=1}^{8} b_i \le 12$ and $b_1 \ge 4$.

- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_0$;
  where $10 \le |b| = \sum_{i=1}^{7} b_i \le 12$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{Z}_{21}^{(b_8)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_1$;
  where $8 \le |b| = \sum_{i=1}^{8} b_i \le 10$ and $b_1 \ge 1$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{Z}_{21}^{(b_8)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$;
  where $9 \le |b| = \sum_{i=1}^{6} b_i \le 11$ and $b_1 \ge 2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(7)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$;
  where $8 \le |b| = \sum_{i=1}^{7} b_i \le 11$ and $b_1 \ge 2$.

∘ In dimension eleven $(\mathcal{X}_{11})$ we have the sum of the following terms:

- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\,\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{D}_{20}\otimes\mathcal{D}_0\otimes\mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(c_1)}y\mathcal{Z}_{32}^{(c_2)}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_0$; where $c_1 + c_2 = 3$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}\mathcal{Z}_{21}^{(b_8)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{12-|b|}\otimes\mathcal{D}_0$;
  where $11 \le |b| = \sum_{i=1}^{8} b_i \le 12$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{Z}_{21}^{(8)}x\mathcal{Z}_{21}^{(b_9)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_1$;
  where $9 \le |b| = \sum_{i=1}^{9} b_i \le 10$ and $b_1 \ge 1$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{Z}_{21}^{(b_8)}x\mathcal{Z}_{21}^{(b_9)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$;
  where $10 \le |b| = \sum_{i=1}^{9} b_i \le 11$ and $b_1 \ge 2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{Z}_{21}^{(b_8)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$;
  where $9 \le |b| = \sum_{i=1}^{8} b_i \le 11$ and $b_1 \ge 2$.

∘ In dimension twelve $(\mathcal{X}_{12})$ we have the sum of the following terms:

- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{D}_{21}\otimes\mathcal{D}_0\otimes\mathcal{D}_0$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{D}_{20}\otimes\mathcal{D}_0\otimes\mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{D}_{21}\otimes\mathcal{D}_0\otimes\mathcal{D}_0$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{Z}_{21}^{(b_8)}x\mathcal{Z}_{21}^{(b_9)}x\mathcal{D}_{10-|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$;
  where $10 \le |b| = \sum_{i=1}^{9} b_i \le 11$ and $b_1 \ge 2$.

Finally, in dimension thirteen $(\mathcal{X}_{13})$ we have:

- $\mathcal{Z}_{32}^{(1)}y\mathcal{Z}_{32}^{(1)}y\mathcal{Z}_{31}^{(1)}z\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(1)}x\mathcal{Z}_{21}^{(1)}x\mathcal{Z}_{21}^{(1)}x\mathcal{Z}_{21}^{(1)}x\mathcal{Z}_{21}^{(1)}x\mathcal{Z}_{21}^{(1)}x\mathcal{Z}_{21}^{(1)}x\mathcal{Z}_{21}^{(1)}x\mathcal{Z}_{21}^{(1)}x\mathcal{D}_{21}\otimes\mathcal{D}_0\otimes\mathcal{D}_0$.

The terms of the Lascoux complex are obtained by the determinantal expansion of the Jacobi-Trudi matrix of the partition [1]. The positions of the terms of the complex are determined by the length of the permutation to which they correspond, [4].

In the case of the partition (9,9,3) we get the following matrix:

$$\begin{bmatrix} \mathcal{D}_9\mathcal{F} & \mathcal{D}_8\mathcal{F} & \mathcal{D}_1\mathcal{F} \\ \mathcal{D}_{10}\mathcal{F} & \mathcal{D}_9\mathcal{F} & \mathcal{D}_2\mathcal{F} \\ \mathcal{D}_{11}\mathcal{F} & \mathcal{D}_{10}\mathcal{F} & \mathcal{D}_3\mathcal{F} \end{bmatrix}$$

Then the Lascoux complex has the correspondence between its terms as pursues:

$\mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \leftrightarrow identity$.
$\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \leftrightarrow (12)$.
$\mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \leftrightarrow (23)$.
$\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \leftrightarrow (123)$.
$\mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \leftrightarrow (132)$.
$\mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \leftrightarrow (13)$.

Thus the resolution of Lascoux in the case of the partition (9,9,3) has the formulation:

$$\mathcal{D}_{11}\mathcal{F}\otimes\mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_1\mathcal{F} \longrightarrow \begin{matrix}\mathcal{D}_{11}\mathcal{F}\otimes\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_2\mathcal{F}\\ \oplus\end{matrix} \longrightarrow \begin{matrix}\mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_3\mathcal{F}\\ \oplus\end{matrix} \longrightarrow \mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_3\mathcal{F}$$

$\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}$ $\qquad$ $\mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$

3. THE CONSEQUENCE

As in [4], we exhibit the terms of the complex (2.1) as:
$\mathcal{X}_0 = \mathcal{L}_0 = \mathcal{M}_0$,
$\mathcal{X}_1 = \mathcal{L}_1 \oplus \mathcal{M}_1$,
$\mathcal{X}_2 = \mathcal{L}_2 \oplus \mathcal{M}_2$,
$\mathcal{X}_3 = \mathcal{L}_3 \oplus \mathcal{M}_3$,
$\mathcal{X}_j = \mathcal{M}_j$; for $j = 4,5,\dots,13$,
where $\mathcal{L}_e$ are the sum of the Lascoux terms and $\mathcal{M}_e$ are the sum of the others.

Now, we define the map $\sigma_1: \mathcal{M}_1 \longrightarrow \mathcal{L}_1$ such that

$$\delta_{\mathcal{L}_1\mathcal{L}_0} \circ \sigma_1 = \delta_{\mathcal{M}_1\mathcal{M}_0} \qquad (2)$$

As follows:

- $\mathcal{Z}_{21}^{(2)}x(v) \longmapsto \frac{1}{2}\, \mathcal{Z}_{21}x\partial_{21}(v)$; where $v \in \mathcal{D}_{11} \otimes \mathcal{D}_7 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x(v) \longmapsto \frac{1}{3}\, \mathcal{Z}_{21}x\partial_{21}^{(2)}(v)$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_6 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)}x(v) \longmapsto \frac{1}{4}\, \mathcal{Z}_{21}x\partial_{21}^{(3)}(v)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_5 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(5)}x(v) \longmapsto \frac{1}{5}\, \mathcal{Z}_{21}x\partial_{21}^{(4)}(v)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(6)}x(v) \longmapsto \frac{1}{6}\, \mathcal{Z}_{21}x\partial_{21}^{(5)}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(7)}x(v) \longmapsto \frac{1}{7}\, \mathcal{Z}_{21}x\partial_{21}^{(6)}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(8)}x(v) \longmapsto \frac{1}{8}\, \mathcal{Z}_{21}x\partial_{21}^{(7)}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(9)}x(v) \longmapsto \frac{1}{8}\, \mathcal{Z}_{21}x\partial_{21}^{(8)}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{32}^{(2)}y(v) \longmapsto \frac{1}{2}\, \mathcal{Z}_{32}y\partial_{32}(v)$; where $v \in \mathcal{D}_9 \otimes \mathcal{D}_{11} \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(3)}y(v) \longmapsto \frac{1}{3}\, \mathcal{Z}_{32}y\partial_{32}^{(2)}(v)$; where $v \in \mathcal{D}_9 \otimes \mathcal{D}_{12} \otimes \mathcal{D}_0$.

It is clear that σ_1 satisfies (4.4.1), then we can define:
$\partial_1: \mathcal{L}_1 \longrightarrow \mathcal{L}_0$ as $\partial_1 = \delta_{\mathcal{L}_1\mathcal{L}_0}$
At this point, we are in a position to define
$\partial_2: \mathcal{L}_2 \longrightarrow \mathcal{L}_1$ by $\partial_2 = \delta_{\mathcal{L}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1}$

Lemma (3.1):

The composition $\partial_1\partial_2$ equal to zero.

Proof:

$$\partial_1\partial_2(a) = \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \left(\delta_{\mathcal{L}_2\mathcal{L}_1}(a) + (\sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1})(a)\right)$$
$$= \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \delta_{\mathcal{L}_2\mathcal{L}_1}(a) + \delta_{\mathcal{L}_1\mathcal{L}_0} \circ (\sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1})(a).$$

But $\delta_{\mathcal{L}_1\mathcal{L}_0} \circ \sigma_1 = \delta_{\mathcal{M}_1\mathcal{M}_0}$ then we get:

$$\partial_1\partial_2(a) = \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \delta_{\mathcal{L}_2\mathcal{L}_1}(a) + \delta_{\mathcal{M}_1\mathcal{M}_0} \circ \delta_{\mathcal{L}_2\mathcal{M}_1}(a).$$

By properties of the boundary map δ we get:
$\partial_1\partial_2 = 0$

We need to define the map $\sigma_2: \mathcal{M}_2 \longrightarrow \mathcal{L}_2$ such that

$$\delta_{\mathcal{M}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1} = \left(\delta_{\mathcal{L}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1}\right) \circ \sigma_2 \qquad (3)$$

As follows:

- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{11} \otimes \mathcal{D}_7 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_6 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_6 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_5 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_5 \otimes \mathcal{D}_3$.

- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_5 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(6)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$
- $\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(7)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(6)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(8)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(8)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(7)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(6)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x(v) \longmapsto \frac{1}{3}\, \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}(v)$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_7 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(4)}x(v) \longmapsto \frac{1}{6}\, \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}(v)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_6 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(5)}x(v) \longmapsto \frac{1}{10}\, \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(3)}(v)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_5 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(6)}x(v) \longmapsto \frac{1}{15}\, \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(4)}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_4 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(7)}x(v) \longmapsto \frac{1}{21}\, \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(5)}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_3 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(8)}x(v) \longmapsto \frac{1}{28}\, \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(6)}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(9)}x(v) \longmapsto \frac{1}{36}\, \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(7)}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(10)}x(v) \longmapsto \frac{1}{45}\, \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y(v) \longmapsto 0$; where $v \in \mathcal{D}_9 \otimes \mathcal{D}_{11} \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(3)}x(v) \longmapsto \frac{1}{3}\, (\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{31}(v) - \mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(2)}(v))$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_8 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(4)}x(v) \longmapsto \frac{1}{12}\, \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{31}(v) - \frac{1}{4}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(3)}(v)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_7 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(5)}x(v) \longmapsto \frac{1}{30}\, \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}(v) - \frac{1}{5}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(4)}(v)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_6 \otimes \mathcal{D}_1$.

- $\mathcal{Z}_{32}^{(2)}\mathcal{Y}\mathcal{Z}_{21}^{(6)}x(v) \longmapsto \frac{1}{60}\ \mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(3)}\partial_{31}(v) - \frac{1}{6}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{31}z\partial_{21}^{(5)}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_5 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{Y}\mathcal{Z}_{21}^{(7)}x(v) \longmapsto \frac{1}{105}\ \mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(4)}\partial_{31}(v) - \frac{1}{7}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{31}z\partial_{21}^{(6)}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{Y}\mathcal{Z}_{21}^{(8)}x(v) \longmapsto \frac{1}{168}\ \mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(5)}\partial_{31}(v) - \frac{1}{8}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{31}z\partial_{21}^{(7)}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{Y}\mathcal{Z}_{21}^{(9)}x(v) \longmapsto \frac{1}{252}\ \mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(6)}\partial_{31}(v) - \frac{1}{9}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{31}z\partial_{21}^{(8)}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{Y}\mathcal{Z}_{21}^{(10)}x(v) \longmapsto \frac{1}{360}\ \mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}(v) - \frac{1}{10}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{31}z\partial_{21}^{(9)}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{Y}\mathcal{Z}_{21}^{(11)}x(v) \longmapsto \frac{1}{495}\ \mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}\partial_{31}(v) - \frac{1}{9}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{31}z\partial_{21}^{(10)}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{Y}\mathcal{Z}_{32}\mathcal{Y}(v) \longmapsto 0$; where $v \in \mathcal{D}_9 \otimes \mathcal{D}_{12} \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{32}^{(2)}\mathcal{Y}(v) \longmapsto 0$; where $v \in \mathcal{D}_9 \otimes \mathcal{D}_{12} \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{Y}\mathcal{Z}_{21}^{(4)}x(v) \longmapsto \frac{1}{3}\ \mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{31}^{(2)}(v) - \frac{1}{6}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v) - \frac{1}{3}\ \mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{31}z\partial_{21}^{(3)}\partial_{32}(v)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_8 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{Y}\mathcal{Z}_{21}^{(5)}x(v) \longmapsto \frac{1}{9}\ \mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{31}^{(2)}(v) - \frac{7}{90}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(3)}\partial_{32}^{(2)}(v) - \frac{2}{9}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{31}z\partial_{21}^{(4)}\partial_{32}(v)$;where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_7 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{Y}\mathcal{Z}_{21}^{(6)}x(v) \longmapsto \frac{1}{18}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}^{(2)}(v) - \frac{2}{45}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(4)}\partial_{32}^{(2)}(v) - \frac{1}{6}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{31}z\partial_{21}^{(5)}\partial_{32}(v)$;where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_6 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{Y}\mathcal{Z}_{21}^{(7)}x(v) \longmapsto \frac{1}{30}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(3)}\partial_{31}^{(2)}(v) - \frac{1}{35}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(5)}\partial_{32}^{(2)}(v) - \frac{2}{15}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{31}z\partial_{21}^{(6)}\partial_{32}(v)$;where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_5 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{Y}\mathcal{Z}_{21}^{(8)}x(v) \longmapsto \frac{1}{45}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(4)}\partial_{31}^{(2)}(v) - \frac{5}{252}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(6)}\partial_{32}^{(2)}(v) - \frac{1}{9}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{31}z\partial_{21}^{(7)}\partial_{32}(v)$;where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_4 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{Y}\mathcal{Z}_{21}^{(9)}x(v) \longmapsto \frac{1}{63}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(5)}\partial_{31}^{(2)}(v) - \frac{11}{756}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(7)}\partial_{32}^{(2)}(v) - \frac{2}{21}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{31}z\partial_{21}^{(8)}\partial_{32}(v)$;where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{Y}\mathcal{Z}_{21}^{(10)}x(v) \longmapsto \frac{1}{84}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(6)}\partial_{31}^{(2)}(v) - \frac{1}{90}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}\partial_{32}^{(2)}(v) - \frac{1}{12}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{31}z\partial_{21}^{(9)}\partial_{32}(v)$;where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{Y}\mathcal{Z}_{21}^{(11)}x(v) \longmapsto \frac{1}{108}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}^{(2)}(v) + \frac{1}{135}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}\partial_{32}\partial_{31}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{Y}\mathcal{Z}_{21}^{(12)}x(v) \longmapsto \frac{1}{135}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}\partial_{31}^{(2)}(v)$; where $v \in \mathcal{D}_{21} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$
- $\mathcal{Z}_{32}^{(2)}\mathcal{Y}\mathcal{Z}_{31}z(v) \longmapsto \frac{1}{3}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{31}z\partial_{32}(v)$; where $v \in \mathcal{D}_{10} \otimes \mathcal{D}_{11} \otimes \mathcal{D}_0$.

Proposition (3.2):

The map σ_2 defined above satisfies (3.2).

Proof:

We can see that for some terms:

- $\left(\delta_{\mathcal{M}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v)\right)$; where $v \in \mathcal{D}_{11} \otimes \mathcal{D}_7 \otimes \mathcal{D}_3$

$= \sigma_1\left(2\mathcal{Z}_{21}^{(2)}x(v)\right) - \mathcal{Z}_{21}x\partial_{21}(v) = \frac{2}{2}\mathcal{Z}_{21}x\partial_{21}(v) - \mathcal{Z}_{21}x\partial_{21}(v) = 0.$

- $\left(\delta_{\mathcal{M}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v)\right)$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_6 \otimes \mathcal{D}_3$

$= \sigma_1\left(3\mathcal{Z}_{21}^{(3)}x(v) - \mathcal{Z}_{21}^{(2)}x\partial_{21}(v)\right) = \mathcal{Z}_{21}x\partial_{21}^{(2)}(v) - \frac{2}{2}\mathcal{Z}_{21}x\partial_{21}^{(2)}(v) = 0.$

- $\left(\delta_{\mathcal{M}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x(v)\right)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$

$= \sigma_1\left(10\mathcal{Z}_{21}^{(5)}x(v) - \mathcal{Z}_{21}^{(2)}x\partial_{21}^{(3)}(v)\right) = 2\ \mathcal{Z}_{21}x\partial_{21}^{(4)}(v) - \frac{4}{2}\mathcal{Z}_{21}x\partial_{21}^{(4)}(v) = 0.$

- $\left(\delta_{\mathcal{M}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(3)}x(v)\right)$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_7 \otimes \mathcal{D}_2$

$= \sigma_1\left(\mathcal{Z}_{21}^{(3)}x\partial_{32}(v) + \mathcal{Z}_{21}^{(2)}x\partial_{31}(v)\right) - \mathcal{Z}_{32}\mathcal{Y}\partial_{21}^{(3)}(v) = \frac{1}{3}\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}(v) + \frac{1}{2}\mathcal{Z}_{21}x\partial_{21}\partial_{31}(v) - \mathcal{Z}_{32}\mathcal{Y}\partial_{21}^{(3)}(v).$

And

$\left(\delta_{\mathcal{L}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1}\right)\left(\frac{1}{3}\mathcal{Z}_{32}\mathcal{Y}\mathcal{Z}_{21}^{(2)}x\partial_{21}(v)\right)$

$= \frac{1}{3}\sigma_1\left(\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{32}(v) + \mathcal{Z}_{21}^{(2)}x\partial_{31}(v)\right) + \frac{1}{3}\mathcal{Z}_{21}x\partial_{31}\partial_{21}(v) - \mathcal{Z}_{32}y\partial_{21}^{(3)}(v)$
$= \frac{1}{3}\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}(v) + \frac{1}{2}\mathcal{Z}_{21}x\partial_{21}\partial_{31}(v) - \mathcal{Z}_{32}y\partial_{21}^{(3)}(v).$

• $\left(\delta_{\mathcal{M}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(10)}x(v)\right)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$
$= \mathcal{Z}_{21}^{(10)}x\partial_{32}(v) + \sigma_1\left(\mathcal{Z}_{21}^{(9)}x\partial_{31}(v)\right) - \mathcal{Z}_{32}y\partial_{21}^{(10)}(v) = \frac{1}{9}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v) - \mathcal{Z}_{32}y\partial_{21}^{(8)}(v).$
And
$\left(\delta_{\mathcal{L}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1}\right)\left(\frac{1}{45}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}(v)\right)$
$= \mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}\partial_{32}(v) + \sigma_1\left(\frac{1}{45}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}(v)\right) + \frac{1}{45}\mathcal{Z}_{21}x\partial_{31}\partial_{21}^{(8)}(v) - \mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{21}^{(10)}(v)$
$= \frac{1}{9}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v) - \mathcal{Z}_{32}y\partial_{21}^{(10)}(v).$

• $\left(\delta_{\mathcal{M}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{32}y\mathcal{Z}_{32}y(v)\right)$; where $v \in \mathcal{D}_9 \otimes \mathcal{D}_{11} \otimes \mathcal{D}_1$
$= \sigma_1\left(2\,\mathcal{Z}_{32}^{(2)}y(v)\right) - \mathcal{Z}_{32}y\partial_{32}(v) = \frac{2}{2}\mathcal{Z}_{32}y\partial_{32}(v) - \mathcal{Z}_{32}y\partial_{32}(v) = 0.$

• $\left(\delta_{\mathcal{M}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(3)}x(v)\right)$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_8 \otimes \mathcal{D}_1$
$= \sigma_1\left(\mathcal{Z}_{21}^{(3)}x\partial_{32}^{(2)}(v) + \mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{31}(v)\right) + \mathcal{Z}_{21}x\partial_{31}^{(2)}(v) - \sigma_1\left(\mathcal{Z}_{32}^{(2)}y\partial_{21}^{(3)}(v)\right)$
$= \frac{1}{3}\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v) + \frac{1}{2}\mathcal{Z}_{21}x\partial_{21}\partial_{32}\partial_{31}(v) + \mathcal{Z}_{21}x\partial_{31}^{(2)}(v) - \frac{1}{2}\mathcal{Z}_{32}y\partial_{32}\partial_{21}^{(3)}(v).$
And
$\left(\delta_{\mathcal{L}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1}\right)\left(\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{31}(v) - \frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(2)}(v)\right)$
$= \sigma_1\left(\frac{1}{3}\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{31}(v)\right) + \frac{1}{3}\mathcal{Z}_{21}x\partial_{31}\partial_{31}(v) - \frac{1}{3}\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{31}(v) - \sigma_1\left(\frac{1}{3}\mathcal{Z}_{32}^{(2)}y\partial_{21}\partial_{21}^{(2)}(v)\right) + \frac{1}{3}\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}^{(2)}(v) +$
$\frac{1}{3}\mathcal{Z}_{32}y\partial_{31}\partial_{21}^{(2)}(v) = \frac{1}{3}\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v) + \frac{1}{2}\mathcal{Z}_{21}x\partial_{21}\partial_{32}\partial_{31}(v) + \mathcal{Z}_{21}x\partial_{31}^{(2)}(v) - \frac{1}{2}\mathcal{Z}_{32}y\partial_{32}\partial_{21}^{(3)}(v).$

• $\left(\delta_{\mathcal{M}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(11)}x(v)\right)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$
$= \mathcal{Z}_{21}^{(11)}x\partial_{31}^{(2)}(v) + \mathcal{Z}_{21}^{(10)}x\partial_{32}\partial_{31}(v) + \sigma_1\left(\mathcal{Z}_{21}^{(9)}x\partial_{31}^{(2)}(v) - \mathcal{Z}_{32}^{(2)}y\partial_{21}^{(11)}(v)\right) = \frac{1}{9}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}^{(2)}(v) - \frac{1}{2}\mathcal{Z}_{32}y\partial_{32}\partial_{21}^{(11)}(v).$
And
$\left(\delta_{\mathcal{L}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1}\right)\left(\frac{1}{495}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}\partial_{31}(v) - \frac{1}{11}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(11)}(v)\right)$
$= \mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}\partial_{32}\partial_{31}(v) + \sigma_1\left(\frac{1}{495}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}^{(2)}(v)\right) + \frac{1}{495}\mathcal{Z}_{21}x\partial_{31}\partial_{21}^{(8)}\partial_{31}(v) -$
$\frac{1}{495}\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{21}^{(8)}\partial_{31}(v) - \sigma_1\left(\frac{1}{11}\mathcal{Z}_{32}^{(2)}y\partial_{21}\partial_{21}^{(10)}(v)\right) + \frac{1}{11}\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}^{(10)}(v) + \frac{1}{11}\mathcal{Z}_{32}y\partial_{31}\partial_{21}^{(10)}(v)$
$= \frac{1}{9}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}^{(2)}(v) - \frac{1}{2}\mathcal{Z}_{32}y\partial_{32}\partial_{21}^{(11)}(v).$

• $\left(\delta_{\mathcal{M}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{32}y(v)\right)$; where $v \in \mathcal{D}_9 \otimes \mathcal{D}_{12} \otimes \mathcal{D}_0$
$= \sigma_1\left(3\,\mathcal{Z}_{32}^{(3)}y(v) - \mathcal{Z}_{32}^{(2)}y\partial_{32}(v)\right) = 0.$

• $\left(\delta_{\mathcal{M}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(4)}x(v)\right)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_5 \otimes \mathcal{D}_0$
$= \sigma_1\left(\mathcal{Z}_{21}^{(4)}x\partial_{32}^{(3)}(v) + \mathcal{Z}_{21}^{(3)}x\partial_{32}^{(2)}\partial_{31}(v) + \mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{31}^{(2)}(v)\right) + \mathcal{Z}_{21}x\partial_{31}^{(3)}(v) - \sigma_1\left(\mathcal{Z}_{32}^{(3)}y\partial_{21}^{(4)}(v)\right)$
$= \frac{1}{4}\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{32}^{(3)}(v) + \frac{1}{3}\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}^{(2)}\partial_{31}(v) + \frac{1}{2}\mathcal{Z}_{21}x\partial_{21}\partial_{32}\partial_{31}^{(2)}(v) +$
$\mathcal{Z}_{21}x\partial_{31}^{(3)}(v) - \frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(4)}\partial_{32}^{(2)}(v) - \frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(3)}\partial_{32}\partial_{31} - \frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{31}^{(2)}(v).$
And
$\left(\delta_{\mathcal{L}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1}\right)\left(\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{31}^{(2)}(v) - \frac{1}{6}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v) - \frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(3)}\partial_{32}(v)\right)$

$= \sigma_1\left(\frac{1}{3}\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{31}^{(2)}(v)\right) + \frac{1}{3}\mathcal{Z}_{21}x\partial_{31}\partial_{31}^{(2)}(v) - \frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{31}^{(2)}(v) -$
$\frac{1}{6}\sigma_1\left(\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}^{(2)}\partial_{32}^{(2)}(v) + \mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{31}\partial_{32}^{(2)}(v)\right) - \frac{1}{6}\mathcal{Z}_{21}x\partial_{31}\partial_{21}^{(2)}\partial_{32}^{(2)}(v) +$
$\mathcal{Z}_{32}y\partial_{21}^{(4)}\partial_{32}^{(2)}(v) - \sigma_1\left(\frac{1}{3}\mathcal{Z}_{32}^{(2)}y\partial_{21}\partial_{21}^{(3)}\partial_{32}(v)\right) + \frac{1}{3}\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}^{(3)}\partial_{32}(v) + \frac{1}{3}\mathcal{Z}_{32}y\partial_{31}\partial_{21}^{(3)}\partial_{32}(v)$
$= \frac{1}{4}\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{32}^{(3)}(v) + \frac{1}{3}\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}^{(2)}\partial_{31}(v) + \frac{1}{2}\mathcal{Z}_{21}x\partial_{21}\partial_{32}\partial_{31}^{(2)}(v) +$
$\mathcal{Z}_{21}x\partial_{31}^{(3)}(v) - \frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(4)}\partial_{32}^{(2)}(v) - \frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(3)}\partial_{32}\partial_{31} - \frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{31}^{(2)}(v).$

- $\left(\delta_{\mathcal{M}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(12)}x(v)\right)$; where $v \in \mathcal{D}_{21} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$

$= \mathcal{Z}_{21}^{(12)}x\partial_{32}^{(3)}(v) + \mathcal{Z}_{21}^{(11)}x\partial_{32}^{(2)}\partial_{31}(v) + \mathcal{Z}_{21}^{(10)}x\partial_{32}\partial_{31}^{(2)}(v) + \sigma_1\left(\mathcal{Z}_{21}^{(9)}x\partial_{31}^{(3)}(v) - \mathcal{Z}_{32}^{(3)}y\partial_{21}^{(12)}(v)\right)$
$= \frac{1}{9}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}^{(3)}(v) - \frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(10)}\partial_{31}^{(2)}(v).$
And
$\left(\delta_{\mathcal{L}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1}\right)\left(\frac{1}{135}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}\partial_{31}^{(2)}(v)\right)$
$= \frac{1}{135}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}\partial_{32}\partial_{31}^{(2)}(v) + \sigma_1\left(\frac{1}{135}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}\partial_{31}^{(2)}(v)\right) + \frac{1}{135}\mathcal{Z}_{21}x\partial_{31}\partial_{21}^{(8)}\partial_{31}^{(2)}(v) - \frac{1}{135}\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{21}^{(8)}\partial_{31}^{(2)}(v)$
$= \frac{1}{9}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}^{(3)}(v) - \frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(10)}\partial_{31}^{(2)}(v).$

- $\left(\delta_{\mathcal{M}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z(v)\right)$; where $v \in \mathcal{D}_{10} \otimes \mathcal{D}_{11} \otimes \mathcal{D}_0$

$= \sigma_1\left(\mathcal{Z}_{32}^{(3)}y\partial_{21}(v)\right) - \mathcal{Z}_{21}x\partial_{32}^{(3)}(v) - \sigma_1\left(\mathcal{Z}_{32}^{(2)}y\partial_{31}(v)\right) = \frac{1}{3}\mathcal{Z}_{32}y\partial_{32}^{(2)}\partial_{21}(v) - \mathcal{Z}_{21}x\partial_{32}^{(3)}(v) - \frac{1}{2}\mathcal{Z}_{32}y\partial_{32}\partial_{31}(v).$
And
$\left(\delta_{\mathcal{L}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1}\right)\left(\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{32}(v)\right)$
$= \sigma_1\left(\frac{1}{3}\mathcal{Z}_{32}^{(2)}y\partial_{21}\partial_{32}(v)\right) - \frac{1}{3}\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{32}(v) - \frac{1}{3}\mathcal{Z}_{32}y\partial_{31}\partial_{32}(v)$
$= \frac{1}{3}\mathcal{Z}_{32}y\partial_{32}^{(2)}\partial_{21}(v) - \mathcal{Z}_{21}x\partial_{32}^{(3)}(v) - \frac{1}{2}\mathcal{Z}_{32}y\partial_{32}\partial_{31}(v)$

Now by employ σ_2 we can also define:
$\partial_3: \mathcal{L}_3 \longrightarrow \mathcal{L}_2$ as $\partial_3 = \delta_{\mathcal{L}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2}$

Lemma (3.3):
The composition $\partial_2\partial_3$ equal to zero.
Proof:
$\partial_2\partial_3(a) = \left(\delta_{\mathcal{L}_2\mathcal{L}_1}(a) + (\sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1})(a)\right) \circ \left(\delta_{\mathcal{L}_3\mathcal{L}_2}(a) + (\sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2})(a)\right)$
$= (\delta_{\mathcal{L}_2\mathcal{L}_1} \circ \delta_{\mathcal{L}_3\mathcal{L}_2})(a) + (\delta_{\mathcal{L}_2\mathcal{L}_1} \circ \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2})(a) + (\sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1} \circ \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2})(a).$
But $\delta_{\mathcal{L}_2\mathcal{L}_1} \circ \sigma_2 + \sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1} \circ \sigma_2 = \delta_{\mathcal{M}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1}$ so we get:
$\partial_2\partial_3(a) = (\delta_{\mathcal{L}_2\mathcal{L}_1} \circ \delta_{\mathcal{L}_3\mathcal{L}_2})(a) + (\delta_{\mathcal{M}_2\mathcal{L}_1} \circ \delta_{\mathcal{L}_3\mathcal{M}_2})(a) + (\sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{L}_1} \circ \delta_{\mathcal{L}_3\mathcal{L}_2})(a)$
$(\sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1} \circ \delta_{\mathcal{L}_2\mathcal{M}_2})(a).$
By properties of the boundary map δ we get:
$\partial_2\partial_3 = 0$

We need the definition of a map $\sigma_3: \mathcal{M}_3 \longrightarrow \mathcal{L}_3$ such that
$$\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2} = \left(\delta_{\mathcal{L}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2}\right) \circ \sigma_3 \quad (4)$$
As follows:
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_6 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_5 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_5 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_5 \otimes \mathcal{D}_3$.

- $\mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21} x \mathcal{Z}_{21}^{(5)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(3)} x\, \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(2)} x\, \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(3)} x\, \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(2)} x\, \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(4)} x\, \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(2)} x\, \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.

- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_{1} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_{1} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_{1} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_{1} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_{1} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_{1} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_{1} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(6)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_{1} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x\,\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(4)}x\,\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(4)}x\,\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x\,\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(2)}x\,\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(3)}x\,\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(3)}x\,\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(2)}x\,\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(5)}x\,\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x\,\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(6)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(1)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(6)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(7)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{3}$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_{7} \otimes \mathcal{D}_{2}$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_{6} \otimes \mathcal{D}_{2}$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_{6} \otimes \mathcal{D}_{2}$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_{5} \otimes \mathcal{D}_{2}$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_{5} \otimes \mathcal{D}_{2}$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_{5} \otimes \mathcal{D}_{2}$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_{4} \otimes \mathcal{D}_{2}$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_{4} \otimes \mathcal{D}_{2}$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_{4} \otimes \mathcal{D}_{2}$.

- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_4 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_3 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_3 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_3 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_3 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_3 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(6)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(8)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(7)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(6)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(9)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(7)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(8)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(8)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(6)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x(v) \longmapsto -\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}(v)$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_8 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(4)}x(v) \longmapsto -\frac{1}{6}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(2)}(v)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_7 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(5)}x(v) \longmapsto -\frac{1}{10}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(3)}(v)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_6 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(6)}x(v) \longmapsto -\frac{1}{15}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(4)}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_5 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(7)}x(v) \longmapsto -\frac{1}{21}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(8)}x(v) \longmapsto -\frac{1}{28}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(9)}x(v) \longmapsto -\frac{1}{36}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(10)}x(v) \longmapsto -\frac{1}{45}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(11)}x(v) \longmapsto -\frac{1}{55}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_7 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_6 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_6 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_5 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_5 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_5 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$.

- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(7)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(6)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(5)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(8)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(7)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(6)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(6)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(5)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(9)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(8)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(7)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(7)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(6)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(6)} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21}^{(5)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(10)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(9)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(8)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(8)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(7)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(7)} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21}^{(6)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(6)} x \mathcal{Z}_{21}^{(5)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{32} y(v) \longmapsto 0$; where $v \in \mathcal{D}_{9} \otimes \mathcal{D}_{12} \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(4)} x(v) \longmapsto \frac{1}{6} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21} \partial_{31}(v) - \frac{1}{4} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(2)} \partial_{32}(v)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_8 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(5)} x(v) \longmapsto \frac{1}{12} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(2)} \partial_{31}(v) - \frac{7}{60} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(3)} \partial_{32}(v)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_7 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(6)} x(v) \longmapsto \frac{1}{20} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(3)} \partial_{31}(v) - \frac{1}{15} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(4)} \partial_{32}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_6 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(7)} x(v) \longmapsto \frac{1}{30} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(4)} \partial_{31}(v) - \frac{3}{70} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(5)} \partial_{32}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_5 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(8)} x(v) \longmapsto \frac{1}{42} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(5)} \partial_{31}(v) - \frac{5}{168} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(6)} \partial_{32}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_4 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(9)} x(v) \longmapsto \frac{1}{56} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(6)} \partial_{31}(v) - \frac{11}{504} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(7)} \partial_{32}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(10)} x(v) \longmapsto \frac{1}{72} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(7)} \partial_{31}(v) - \frac{1}{60} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(8)} \partial_{32}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(11)} x(v) \longmapsto \frac{1}{90} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(8)} \partial_{31}(v) + \frac{1}{110} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(9)} \partial_{32}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(12)} x(v) \longmapsto \frac{1}{110} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(9)} \partial_{31}(v)$; where $v \in \mathcal{D}_{21} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(4)} x(v) \longmapsto -\frac{1}{3} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(2)} \partial_{32}(v)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_8 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(5)} x(v) \longmapsto -\frac{1}{6} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(3)} \partial_{32}(v)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_7 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(6)} x(v) \longmapsto -\frac{1}{10} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(4)} \partial_{32}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_6 \otimes \mathcal{D}_0$.

- $\mathcal{Z}_{32}y\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(7)}x(v) \longmapsto -\frac{1}{15}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_{5} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(8)}x(v) \longmapsto -\frac{1}{21}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_{4} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(9)}x(v) \longmapsto -\frac{1}{28}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{3} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(10)}x(v) \longmapsto -\frac{1}{36}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{32}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_{2} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(11)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_{1} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(12)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{21} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v) \longmapsto -\frac{1}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{31}(v) - \frac{1}{18}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{32}(v)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_{7} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x(v) \longmapsto -\frac{1}{18}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{31}(v) - \frac{1}{45}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{32}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_{6} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto -\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{31}(v) - \frac{1}{6}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{32}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_{6} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}x(v) \longmapsto -\frac{1}{30}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}(v) - \frac{1}{90}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_{5} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto -\frac{2}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}(v) - \frac{4}{45}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_{5} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto -\frac{2}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}(v) - \frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_{5} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}x(v) \longmapsto -\frac{1}{45}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{31}(v) - \frac{2}{315}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_{4} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto -\frac{1}{6}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{31}(v) - \frac{1}{18}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_{4} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto -\frac{5}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{31}(v) - \frac{2}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_{4} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto -\frac{-5}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{31}(v) - \frac{2}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_{4} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(8)}x\mathcal{Z}_{21}x(v) \longmapsto -\frac{1}{63}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) - \frac{1}{252}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{3} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto -\frac{14}{105}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) - \frac{4}{105}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{3} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto -\frac{1}{2}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) - \frac{1}{6}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{3} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto -\frac{10}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) - \frac{4}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{3} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto -\frac{5}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) - \frac{5}{6}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_{3} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(9)}x\mathcal{Z}_{21}x(v) \longmapsto -\frac{1}{84}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}(v) - \frac{1}{378}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{32}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_{2} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(8)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto -\frac{1}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}(v) - \frac{1}{36}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{32}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_{2} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto -\frac{7}{15}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}(v) - \frac{2}{15}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{32}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_{2} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto -\frac{7}{6}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}(v) - \frac{7}{18}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{32}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_{2} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(6)}x(v) \longmapsto -\frac{7}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}(v) - \frac{7}{6}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{32}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_{2} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto -\frac{35}{18}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}(v) - \frac{7}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{32}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_{2} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(9)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto -\frac{2}{21}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v) - \frac{3}{7}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{32}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_{1} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(8)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto -\frac{4}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v) - \frac{4}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{32}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_{1} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(7)}x(v) \longmapsto -\frac{28}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v) - 4\,\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{32}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_{1} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto -\frac{56}{5}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v) - \frac{14}{5}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{32}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_{1} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(6)}x(v) \longmapsto -\frac{28}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v) - \frac{14}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{32}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_{1} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto -\frac{7}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v) - \frac{21}{5}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{32}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_{1} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(10)}x\mathcal{Z}_{21}x(v) \longmapsto -\frac{1}{108}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v) - \frac{1}{12}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{32}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_{1} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(9)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto -\frac{4}{14}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{31}(v)$; where $v \in \mathcal{D}_{21} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(8)}x(v) \longmapsto -\frac{24}{6}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{31}(v)$; where $v \in \mathcal{D}_{21} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(8)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto -\frac{8}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{31}(v)$; where $v \in \mathcal{D}_{21} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{0}$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(7)}x(v) \longmapsto -\frac{14}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{31}(v)$; where $v \in \mathcal{D}_{21} \otimes \mathcal{D}_{0} \otimes \mathcal{D}_{0}$.

- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto -\frac{14}{5}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{31}(v)$; where $v \in \mathcal{D}_{21} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(6)}x(v) \longmapsto -\frac{21}{5}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{31}(v)$; where $v \in \mathcal{D}_{21} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(10)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto -\frac{1}{12}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{31}(v)$; where $v \in \mathcal{D}_{21} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(11)}x\mathcal{Z}_{21}x(v)(v) \longmapsto 0$; where $v \in \mathcal{D}_{21} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(2)}x(v) \longmapsto \frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}(v)$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_8 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(3)}x(v) \longmapsto \frac{1}{6}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(2)}(v)$;where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_7 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(4)}x(v) \longmapsto \frac{1}{10}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(3)}(v)$;where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_6 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(5)}x(v) \longmapsto \frac{1}{15}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(4)}(v)$;where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_5 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(6)}x(v) \longmapsto \frac{1}{21}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}(v)$;where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(7)}x(v) \longmapsto \frac{1}{28}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}(v)$;where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(8)}x(v) \longmapsto \frac{1}{36}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}(v)$;where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(9)}x(v) \longmapsto \frac{1}{45}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}(v)$;where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(10)}x(v) \longmapsto \frac{1}{55}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}(v)$;where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z(v) \longmapsto 0$;where $v \in \mathcal{D}_{10} \otimes \mathcal{D}_{11} \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(2)}x(v) \longmapsto \frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{31}(v)$;where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_9 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(3)}x(v) \longmapsto \frac{1}{6}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}\partial_{31}(v) - \frac{1}{12}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}(v)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_8 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(4)}x(v) \longmapsto \frac{1}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{31}(v) - \frac{7}{90}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{32}(v)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_7 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(5)}x(v) \longmapsto \frac{1}{12}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{31}(v) - \frac{1}{15}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{32}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_6 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(6)}x(v) \longmapsto \frac{1}{15}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}(v) - \frac{2}{35}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_5 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(7)}x(v) \longmapsto \frac{1}{18}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{31}(v) - \frac{25}{504}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_4 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(8)}x(v) \longmapsto \frac{1}{21}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) + \frac{11}{252}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(9)}x(v) \longmapsto \frac{7}{168}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}(v) + \frac{11}{252}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{32}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(10)}x(v) \longmapsto \frac{1}{27}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v) + \frac{4}{165}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{32}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(11)}x(v) \longmapsto \frac{1}{30}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{31}(v)$;where $v \in \mathcal{D}_{21} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$.

Proposition (3.4):

The map σ_3 defined above satisfies (3.3).

Proof: We can see that for some terms:

- $\left(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v)\right)$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_6 \otimes \mathcal{D}_3$

$= \sigma_2\left(2\,\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v) - 2\,\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x(v) + \mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{21}(v)\right) = 0.$

- $\left(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x(v)\right)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$

$= \sigma_2\left(4\,\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(2)}x(v) - 10\,\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(5)}x(v) + \mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x\partial_{21}^{(2)}(v)\right) = 0.$

- $\left(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x(v)\right)$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_8 \otimes \mathcal{D}_1$

$= \sigma_2\left(2\,\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(3)}x(v) - \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x\partial_{32}(v)\right) - \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{31}(v) + \sigma_2\left(\mathcal{Z}_{32}y\mathcal{Z}_{32}y\partial_{21}^{(3)}(v)\right)$

$= -\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{31}(v) - \frac{2}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(2)}(v) - \frac{1}{3}\,\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{32}(v).$

And

$\left(\delta_{\mathcal{L}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2}\right)\left(-\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}(v)\right)$

$= \sigma_2\left(\frac{1}{3}\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}(v)\right) - \frac{1}{3}\,\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}(v) + \quad \sigma_2\left(\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{21}(v)\right) - \frac{2}{3}\,\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(2)}(v)$

$= -\frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{31}(v) - \frac{2}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(2)}(v) - \frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{32}(v).$

• $\left(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(11)}x(v)\right)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$
$= \sigma_2\left(2\,\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(11)}x(v)\right) - \mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(11)}x\partial_{32}(v) - \sigma_2\left(\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(10)}x\partial_{31}(v) + \mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(11)}(v)\right)$
$= -\frac{1}{55}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}\partial_{31}(v) - \frac{2}{11}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(10)}(v).$
And
$\left(\delta_{\mathcal{L}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2}\right)\left(-\frac{1}{55}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}(v)\right)$
$= \sigma_2\left(\frac{1}{55}\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}^{(9)}(v)\right) - \frac{1}{55}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}^{(9)}(v) + \sigma_2\left(\frac{1}{55}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(2)}\partial_{21}^{(9)}(v)\right) - \frac{10}{55}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(10)}(v)$
$= -\frac{1}{55}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}\partial_{31}(v) - \frac{2}{11}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(10)}(v).$

• $\left(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x(v)\right)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_7 \otimes \mathcal{D}_1$
$= \sigma_2\left(\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x\partial_{32}^{(2)}(v) + \mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x\partial_{32}\partial_{31}(v) + \mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{31}^{(2)}(v) - 4\,\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x(v) + \mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(3)}x\partial_{21}(v)\right) = 0.$

• $\left(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}(v)\right)$; where $v \in \mathcal{D}_9 \otimes \mathcal{D}_{12} \otimes \mathcal{D}_0$
$= \sigma_2\left(2\,\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}(v) - 2\,\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}^{(2)}\mathcal{y}(v) + \mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{32}(v)\right) = 0.$

• $\left(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(4)}x(v)\right)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_8 \otimes \mathcal{D}_0$
$= \sigma_2\left(3\,\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x(v) - \mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\partial_{32}(v) - \mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(3)}x\partial_{31}(v) + \mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(4)}(v)\right)$
$= \frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{31}^{(2)}(v) - \frac{1}{2}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v) - \frac{3}{4}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(3)}\partial_{32}(v) - \frac{1}{12}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{32}\partial_{31}(v) +$
$\frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(2)}\partial_{31}(v).$
And
$\left(\delta_{\mathcal{L}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2}\right)\left(\frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}\partial_{31}(v) - \frac{1}{4}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}(v)\right)$
$= \sigma_2\left(-\frac{1}{6}\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}\partial_{31}(v)\right) + \frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}\partial_{31}(v) + \sigma_2\left(\frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(2)}\partial_{21}\partial_{31}(v)\right) +$
$\frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}\partial_{21}\partial_{31}(v) +$
$\sigma_2\left(\frac{1}{4}\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}^{(2)}\partial_{32}(v)\right) - \frac{1}{4}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}^{(2)}\partial_{32}(v) +$
$\sigma_2\left(\frac{1}{4}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(2)}\partial_{21}^{(2)}\partial_{32}(v)\right) - \frac{1}{4}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}\partial_{21}^{(2)}\partial_{32}(v)$
$= \frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{31}^{(2)}(v) - \frac{1}{2}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v) - \frac{3}{4}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(3)}\partial_{32}(v) - \frac{1}{12}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{32}\partial_{31}(v) +$
$\frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(2)}\partial_{31}(v).$

• $\left(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(12)}x(v)\right)$; where $v \in \mathcal{D}_{21} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$
$= \sigma_2\left(3\,\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(12)}x(v)\right) - \mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(12)}x\partial_{32}(v) - \sigma_2\left(\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(11)}x\partial_{31}(v) + \mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(12)}(v)\right)$
$= \frac{1}{55}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}\partial_{31}^{(2)}(v) + \frac{1}{11}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(10)}\partial_{31}(v).$
And
$\left(\delta_{\mathcal{L}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2}\right)\left(\frac{1}{110}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{31}(v)\right)$
$= \sigma_2\left(-\frac{1}{110}\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}^{(9)}\partial_{31}(v)\right) + \frac{1}{110}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}^{(9)}\partial_{31}(v) -$
$\sigma_2\left(\frac{1}{110}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(2)}\partial_{21}^{(9)}\partial_{31}(v)\right) + \frac{1}{110}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}\partial_{21}^{(9)}\partial_{31}(v) = \frac{1}{55}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}\partial_{31}^{(2)}(v) +$
$\frac{1}{11}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(10)}\partial_{31}(v).$

• $\left(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}y\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(4)}x(v)\right)$; where $v\in\mathcal{D}_{13}\otimes\mathcal{D}_8\otimes\mathcal{D}_0$

$=\sigma_2\big(3\,\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(4)}x(v)-\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(4)}x\partial_{32}^{(2)}(v)-\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x\partial_{32}\partial_{31}(v)-$
$\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{31}^{(2)}(v)+\mathcal{Z}_{32}y\,\mathcal{Z}_{32}^{(2)}y\partial_{21}^{(4)}(v)\big)$

$=-\frac{2}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v)-\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(3)}\partial_{32}(v)-\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{32}\partial_{31}(v).$

And

$\left(\delta_{\mathcal{L}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{L}_3\mathcal{M}_2}\right)\left(-\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}(v)\right)$

$=\sigma_2\left(\frac{1}{3}\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}^{(2)}\partial_{32}(v)\right)-\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}^{(2)}\partial_{32}(v)+$
$\sigma_2\left(\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{21}^{(2)}\partial_{32}(v)\right)-\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}\partial_{21}^{(2)}\partial_{32}(v)$

$=-\frac{2}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v)-\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(3)}\partial_{32}(v)-\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{32}\partial_{31}(v).$

• $\left(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}y\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(12)}x(v)\right)$; where $v\in\mathcal{D}_{21}\otimes\mathcal{D}_0\otimes\mathcal{D}_0$

$=\sigma_2\left(3\,\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(12)}x(v)\right)-\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(12)}x\partial_{32}^{(2)}(v)-\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(11)}x\partial_{32}\partial_{31}(v)-$
$\sigma_2\big(-\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(10)}x\partial_{31}^{(2)}(v)+\mathcal{Z}_{32}y\,\mathcal{Z}_{32}^{(2)}y\partial_{21}^{(12)}(v)\big)=\frac{3}{135}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}\partial_{31}^{(2)}(v)-\frac{1}{45}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}\partial_{31}^{(2)}(v)=0.$

• $\left(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v)\right)$; where $v\in\mathcal{D}_{14}\otimes\mathcal{D}_7\otimes\mathcal{D}_0$

$=\sigma_2\big(\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x\partial_{32}^{(2)}(v)+\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{31}(v)+\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x\partial_{32}\partial_{31}^{(2)}(v)+$
$\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{31}^{(3)}(v)-5\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(5)}x(v)+\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(4)}x\partial_{21}(v)\big)$

$=-\frac{2}{9}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{31}^{(2)}(v)-\frac{1}{9}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(3)}\partial_{32}^{(2)}(v)-\frac{2}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(4)}\partial_{32}(v)$
$-\frac{1}{6}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}\partial_{31}(v)-\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(3)}\partial_{31}(v).$

And

$\left(\delta_{\mathcal{L}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{L}_3\mathcal{M}_2}\right)\left(\frac{-1}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{31}(v)-\frac{1}{18}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{32}(v)\right)$

$=\sigma_2\left(\frac{1}{9}\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}^{(2)}\partial_{31}(v)\right)-\frac{1}{9}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}^{(2)}\partial_{31}(v)+$
$\sigma_2\left(\frac{1}{9}\mathcal{Z}_{32}y\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{21}^{(2)}\partial_{31}(v)\right)-\frac{1}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}\partial_{21}^{(2)}\partial_{31}(v)+$
$\sigma_2\left(\frac{1}{18}\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}^{(3)}\partial_{32}(v)\right)-\frac{1}{18}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}^{(3)}\partial_{32}(v)+$
$\sigma_2\left(\frac{1}{18}\mathcal{Z}_{32}y\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{21}^{(3)}\partial_{32}(v)\right)-\frac{1}{18}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}\partial_{21}^{(3)}\partial_{32}(v)$

$=-\frac{2}{9}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{31}^{(2)}(v)-\frac{1}{9}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(3)}\partial_{32}^{(2)}(v)-\frac{2}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(4)}\partial_{32}(v)$
$-\frac{1}{6}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}\partial_{31}(v)-\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(3)}\partial_{31}(v).$

• $\left(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(11)}x\mathcal{Z}_{21}x(v)\right)$; where $v\in\mathcal{D}_{21}\otimes\mathcal{D}_0\otimes\mathcal{D}_0$

$=\mathcal{Z}_{21}^{(11)}x\mathcal{Z}_{21}x\partial_{32}^{(3)}(v)+\mathcal{Z}_{21}^{(10)}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{31}(v)+\mathcal{Z}_{21}^{(9)}x\mathcal{Z}_{21}x\partial_{32}\partial_{31}^{(2)}(v)+$
$\sigma_2\big(\mathcal{Z}_{21}^{(8)}x\mathcal{Z}_{21}x\partial_{31}^{(3)}(v)-12\,\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(10)}x(v)+\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(11)}x\partial_{21}(v)\big)=0.$

• $\left(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(2)}x(v)\right)$; where $v\in\mathcal{D}_{12}\otimes\mathcal{D}_8\otimes\mathcal{D}_1$

$=\sigma_2\big(\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(3)}x(v)-\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x\partial_{32}^{(2)}(v)+\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x\partial_{32}(v)-$
$\mathcal{Z}_{32}y\mathcal{Z}_{32}y\partial_{21}^{(3)}(v)\big)+\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(2)}(v)=\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{31}(v)+\frac{2}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(2)}(v)+\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{32}(v).$

And

$\left(\delta_{\mathcal{L}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{L}_3\mathcal{M}_2}\right)\left(\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}(v)\right)$

$= \sigma_2\left(-\frac{1}{3}\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}(v)\right)+\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}(v)-\sigma_2\left(\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{21}(v)\right)+\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}\partial_{21}(v)$
$= \frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{31}(v)+\frac{2}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(2)}(v)+\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{32}(v).$

• $(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2})(\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z(v))$; where $v\in\mathcal{D}_{10}\otimes\mathcal{D}_{11}\otimes\mathcal{D}_0$
$= \sigma_2\left(2\,\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z(v)-2\,\mathcal{Z}_{32}y\mathcal{Z}_{31}z(v)+\mathcal{Z}_{32}y\mathcal{Z}_{32}y\partial_{31}(v)\right)=0.$

• $(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2})\left(\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(2)}x(v)\right)$; where $v\in\mathcal{D}_{12}\otimes\mathcal{D}_6\otimes\mathcal{D}_0$
$= \sigma_2\left(-\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x\partial_{32}^{(2)}(v)+\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x\partial_{32}(v)+\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{32}y\partial_{21}^{(3)}(v)+\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(2)}(v)\right)$
$= \frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{31}(v)+\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}\partial_{31}(v).$
And
$(\delta_{\mathcal{L}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{L}_3\mathcal{M}_2})\left(\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{31}(v)\right)$
$= \sigma_2\left(-\frac{1}{3}\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{31}(v)\right)+\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{31}(v)-\sigma_2\left(\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{31}(v)\right)+\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}\partial_{31}(v)$
$= \frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{31}(v)+\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}\partial_{31}(v).$

• $(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2})\left(\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(11)}x(v)\right)$; where $v\in\mathcal{D}_{21}\otimes\mathcal{D}_0\otimes\mathcal{D}_0$
$= \sigma_2\left(9\,\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(12)}x(v)\right)-\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(11)}x\partial_{32}^{(3)}(v)+\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(12)}x\partial_{32}-\sigma_2\left(\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{32}y\partial_{21}^{(12)}(v)+\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\partial_{21}^{(11)}(v)\right)$
$= \frac{1}{15}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}\partial_{31}^{(2)}(v)+\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(10)}\partial_{31}(v).$
And
$(\delta_{\mathcal{L}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{L}_3\mathcal{M}_2})\left(\frac{1}{30}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(9)}\partial_{31}(v)\right)$
$= \sigma_2\left(-\frac{1}{30}\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}^{(9)}\partial_{31}(v)\right)+\frac{1}{30}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}^{(9)}\partial_{31}(v)-$
$\sigma_2\left(\frac{1}{30}\mathcal{Z}_{32}y\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{21}^{(9)}\partial_{31}(v)\right)+\frac{1}{24}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}\partial_{21}^{(9)}\partial_{31}(v)$
$= \frac{1}{15}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(8)}\partial_{31}^{(2)}(v)+\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(10)}\partial_{31}(v).$

Eventually, we define the boundary maps in the complex:

$$0\longrightarrow\mathcal{L}_3\xrightarrow{\partial_3}\mathcal{L}_2\xrightarrow{\partial_2}\mathcal{L}_1\xrightarrow{\partial_1}\mathcal{L}_0\,; \tag{5}$$

where ∂_1 is the operation of indicated polarization operators, ∂_1, ∂_2 and ∂_3 defined as follows:
• $\partial_1(\mathcal{Z}_{21}x(v))=\partial_{21}(v)$; where $v\in\mathcal{D}_{10}\otimes\mathcal{D}_8\otimes\mathcal{D}_3$.
• $\partial_1(\mathcal{Z}_{32}y(v))=\partial_{32}(v)$; where $v\in\mathcal{D}_9\otimes\mathcal{D}_{10}\otimes\mathcal{D}_2$.
• $\partial_2(\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x(v))=\frac{1}{2}\mathcal{Z}_{21}x\partial_{21}\partial_{32}(v)+\mathcal{Z}_{21}x\partial_{31}(v)-\mathcal{Z}_{32}y\partial_{21}^{(2)}(v)$; where $v\in\mathcal{D}_{11}\otimes\mathcal{D}_8\otimes\mathcal{D}_2$.
• $\partial_2(\mathcal{Z}_{32}y\mathcal{Z}_{31}z(v))=\frac{1}{2}\mathcal{Z}_{32}y\partial_{32}\partial_{21}(v)-\mathcal{Z}_{21}x\partial_{32}^{(2)}(v)-\mathcal{Z}_{32}y\partial_{31}(v)$; where $v\in\mathcal{D}_{10}\otimes\mathcal{D}_{10}\otimes\mathcal{D}_1$.
• $\partial_3(\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x(v))=\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{32}(v)+\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}(v)$; where $v\in\mathcal{D}_{11}\otimes\mathcal{D}_9\otimes\mathcal{D}_1$.

Theorem (3.5):
The complex (3.4) is exact and in characteristic-zero gives a resolution of $K_{(9,9,3)}(\mathcal{F})$.
Proof:
First, we prove the exactness of the complex

$$0\longrightarrow\mathcal{L}_3\xrightarrow{\partial_3}\mathcal{L}_2\xrightarrow{\partial_2}\mathcal{L}_1.$$

Since one component of the map ∂_3 is a diagonalization of $\mathcal{D}_2$ into $\mathcal{D}_1\otimes\mathcal{D}_1$ it is clear that ∂_3 is injective. To prove the exactness at $\mathcal{L}_2$.
For this, we need to show that:
If $v\in\ker(\partial_2)$ then $\exists\ w\in\mathcal{L}_3$ such that $\partial_3(w)=v$.
If $\partial_2(v)=0$ then $\exists\ (a,b)\in\mathcal{L}_3\oplus\mathcal{M}_3$ such that

$\delta(a,b) = (v,0) \in \mathcal{L}_2 \oplus \mathcal{M}_2$, but
$\delta(a,b) = \delta_{\mathcal{L}_3\mathcal{L}_2}(a) + \delta_{\mathcal{L}_3\mathcal{M}_2}(a) + \delta_{\mathcal{M}_2\mathcal{L}_2}(b) + \delta_{\mathcal{M}_3\mathcal{M}_2}(b)$. So we get:

$$\delta_{\mathcal{L}_3\mathcal{L}_2}(a) + \delta_{\mathcal{M}_3\mathcal{L}_2}(b) = v \quad (6)$$

and

$$\delta_{\mathcal{L}_3\mathcal{M}_2}(a) + \delta_{\mathcal{M}_3\mathcal{M}_2}(b) = 0 \quad (7)$$

Now if $w = a + \sigma_3(b)$ we can see that $\partial_3(w) = v$ in fact
$\partial_3(a) = \delta_{\mathcal{L}_3\mathcal{L}_2}(a) + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2}(a)$, and
$\partial_3(\sigma_3(b)) = \delta_{\mathcal{M}_3\mathcal{L}_2}(b) + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2}(b)$, so

$$\begin{aligned}\partial_3(a + \sigma_3(b)) &= \partial_3(a) + \partial_3(\sigma_3(b)) \\ &= \delta_{\mathcal{L}_3\mathcal{L}_2}(a) + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2}(a) + \delta_{\mathcal{M}_2\mathcal{L}_2}(b) + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2}(b) \\ &= \delta_{\mathcal{L}_3\mathcal{L}_2}(a) + \delta_{\mathcal{M}_3\mathcal{L}_2}(b) + \sigma_2 \circ \left((\delta_{\mathcal{L}_3\mathcal{M}_2}(a) + \delta_{\mathcal{M}_3\mathcal{M}_2}(b)\right).\end{aligned}$$

Hence from (1) and (2), we get $\partial_3(w) = v$; where $w = a + \partial_3(b)$.
This proves the exactness at $\mathcal{L}_2$.

As the same way we can prove the exactness at $\mathcal{L}_1$.
Finally, we get the complex:

$$0 \longrightarrow \mathcal{L}_3 \xrightarrow{\partial_3} \mathcal{L}_2 \xrightarrow{\partial_2} \mathcal{L}_1 \xrightarrow{\partial_1} \mathcal{L}_0 \longrightarrow \mathcal{K}_{(9,9,3)}(\mathcal{F}) \longrightarrow 0,$$

is exact.

Conflicts Of Interest

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