Babylonian Journal of Mathematics
Vol. **2024**, **pp**. 171–177
DOI:https://doi.org/10.58496/BJM/2024/018; ISSN:3006-113X
https://mesopotamian.press/journals/index.php/mathematics

Research Article

Factor Group for Some $\mathcal{SL}$ groups

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ARTICLE INFO

Article History

Received 18 Sep 2024
Revised: 18 Oct 2024
Accepted 16 Nov 2024
Published 08 Dec 2024

Keywords

circular segmentation

class functions

special linear group

finite group

invertible n × n matrices

ABSTRACT

This work found the circular segmentation (CS) for $p = 11, 13, 17$, and 19 to $\mathcal{SL}$ $(2, p)$, after we find the (RRCT) for each group and diagonal the matrix of this (RRCT) if we suppose that the terms of this basic diagonal are $a, b, c, \ldots, n$ then the (CS) is $\mathcal{Z}_a \oplus \mathcal{Z}_b \oplus \mathcal{Z}_c \oplus \ldots \oplus \mathcal{Z}_n$.

Let V vector void on F, $\mathcal{GL}$(V) indicate whole linear isomo. of V upon same, a representation of G for representation void V is a homo. of G to $\mathcal{GL}$(V). A representation model of G is a homo. of G to $\mathcal{GL}(\mathrm{n}, \mathrm{F})$, where n is the degree of the representation model.

Whole invertible $n \times n$ model form a group on a field F indicate $\mathcal{GL}(n,\mathrm{F})$. A homo. of GL(n,F) to F-{0} is the determinant of these model, $\mathcal{SL}(n,\mathrm{F})$ indicate the kernel of it. Thus (n,F) is a subgp. of $\mathcal{GL}(n,\mathrm{F})$ include whole models of determinant 1 on F.

1. INTRODUCTION

The collection of whole $\mathcal{Z}$-account grade maps respecting a restricted group G of commutative group cf (G,Z) beneath spot wise addendum. Into this one group own $\mathcal{Z}$ - account generalized characters a subgroup indicate R(G). The significance of character and representation notion for survey of group's proceeds on one duke to the reality ought to be indispensable to offer a fixed depiction of a group; it can accomplish together with a model representation. Another duke, group notion profit at most to the employ of characters representations and, while these oncoming are utilize as a further to resolve the constructing a group. Furthermore character and representation notion supply diverse applications, not exclusive in other offshoot of mathematics but as well in chemistry and physics, [1].
We compute the circular segmentation (CS) for $p = 11, 13, 17$, and 19 to $\mathcal{SL}$ $(2, p)$ from the rational representations character table (RRCT).

2. FUNDAMENTAL

Theorem 2.1: [2]
The group has order $p^k\ (p^{2k} - 1)$.

Theorem 2.2: [2]
The conjugacy classes is satisfied by the following table From [3]

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TABLE I. THE CONJUGACY CLASSES IS SATIFIED

g ∈ G	Notation	C_g	$\lvert C_g \rvert$	$\lvert C_G(g) \rvert$
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	1	C_1	1	$p^k(p^{2k}-1)$
$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	z	C_z	1	$p^k(p^{2k}-1)$
$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$	c	C_c	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} 1 & 0 \\ \nu & 1 \end{pmatrix}$	d	C_d	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}$	zc	C_{zc}	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} -1 & 0 \\ -\nu & -1 \end{pmatrix}$	zd	C_{zd}	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} \nu^{\ell} & 0 \\ 0 & \nu^{-\ell} \end{pmatrix}$	a^{ℓ}	Ca^{ℓ}	$p^k(p^k+1)$	p^k-1
Element of order(p^k+1) m	b^m	Cb^m	$p^k(p^k-1)$	p^k+1

$$\mu(n)=\begin{cases} 1 & \text{if } n=1 \\ 0 & \text{if } a^2/n \text{ for some } a>1 \\ (-1)^K & \text{if } n=p_1p_2\cdots p_K,\ p_i \text{ are distinct primes.} \end{cases}$$

<u>Lemma (2.3):</u> [1].

For G = $\mathcal{SL}(2, p^k)$, $e, e' < (p^k-1)/2$ and $f, f' < (p^k+1)/2$, also $\varepsilon=(-1)^{(p^k-1)/2}$, let ρ, $\sigma \in$ C are (p^k-1) and (p^k+1)-th origin of 1 resp.

$\mathcal{B}(k)=\begin{cases}1 & \text{if } k \text{ is even}\\ 2 & \text{otherwise}\end{cases}$, $E(p^k)=\begin{cases}1 & \text{if } p^k\ i\equiv 3 \bmod 4\\ 2 & \text{otherwise}\end{cases}$, $\mathcal{A}(e)=(1/2)\varnothing((p^k-1)/e)$,

$\mathcal{C}(f)=(1/2)\varnothing((p^k+1)/f)$, $\tau_1(e,e')=[\varnothing((p^k-1)/e)/\varnothing((p^k-1)/ee')]\,\mu((p^k-1)/ee')$, $\tau_2(f,f')=[\varnothing((p^k+1)/f)/\varnothing((p^k+1)/ff')]\,\mu((p^k+1)/ff')$

The (CTRR) is

TABLE II. THE CTRR.

Cg	1	z	c and d	$a^{e'}$	$b^{f'}$
| Cg |	1	1	$(p^{2k}-1)/2$	$p^k(p^k+1)$	$p^k(p^k-1)$
|CG(g)|	$p^k(p^{2k}-1)$	$p^k(p^{2k}-1)$	$2p^k$	p^k-1	p^k+1
1_G	1	1	1	1	1
ψ	p^k	p^k	0	1	-1
χ_e	$(p^k+1)A(e)B(e)$	$(-1)^e(p^k+1)A(e)B(e)$	$A(e)B(e)$	$B(e)\tau_1(e,e')$	0
θ_f	$(p^k-1)C(f)B(f)$	$(-1)^f(p^k-1)C(f)B(f)$	$-C(f)B(f)$	0	$-B(f)\tau_2(f,f')$
$\zeta_1+\zeta_2$	(p^k+1)	$\varepsilon(p^k+1)$	1	$(-1)^{e'}2$	0
$\eta_1+\eta_1$	$(p^k-1)E(p^k)$	$-\varepsilon(p^k-1)E(p^k)$	-1	0	$(-1)^{f'+1}2E(p^k)$

<u>Theorem (2.4):</u> [4,5]

$$K(G) = \bigoplus_{i=1}^{n} Z\,P^i .$$

3. THE RESULTS

3.1 The (CS) of $\mathcal{SL}(2,11)$

The (RRCT) is:

TABLE III. THE RRCT

C_g	1	z	c	zc	a	a^2	b	b^2	b^3	b^4
| C_g |	1	1	60	60	132	132	110	110	110	110
| $C_G(g)$|	1320	1320	22	22	10	10	12	12	12	12
1_G	1	1	1	1	1	1	1	1	1	1
ψ	11	11	0	0	1	1	-1	-1	-1	-1
χ_1	24	-24	2	-2	1	-1	0	0	0	0
χ_2	24	24	2	2	-1	-1	0	0	0	0
θ_1	20	-20	-2	2	0	0	0	-2	0	2
θ_2	10	10	-1	-1	0	0	-1	1	2	1
θ_3	10	-10	-1	1	0	0	0	2	0	-2
θ_4	10	10	-1	-1	0	0	1	1	-2	1
$\xi_1+\xi_2$	12	-12	1	-1	-2	2	0	0	0	0
$\eta_1+\eta_2$	10	10	-1	-1	0	0	2	-2	2	-2

Then the diagonal of it is

$$\begin{pmatrix} 1320 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -330 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Thus, we obtained $K(\mathcal{SL}(2,11)) = \oplus\mathbb{Z}_{\wp}$ for $\wp = 1320,330,2,1,1,2,2,6,2,1$.

3.2 The (CS) of $\mathcal{SL}(2,13)$

The (RRCT) is :

C_g	1	z	c	zc	a	a^2	a^3	a^4	b	b^2
$\lvert C_g \rvert$	1	1	84	84	182	182	182	182	156	156
$\lvert C_G(g) \rvert$	2184	2184	26	26	12	12	12	12	14	14
1_G	1	1	1	1	1	1	1	1	1	1
ψ	13	13	0	0	1	1	1	1	-1	-1
χ_1	28	-28	2	-2	0	2	0	-2	0	0
χ_2	14	14	1	1	1	-1	-2	-1	0	0
χ_3	14	-14	1	-1	0	-2	0	2	0	0
χ_4	14	14	1	1	-1	-1	2	-1	0	0
θ_1	36	-36	-3	3	0	0	0	0	-1	1
θ_2	36	36	-3	-3	0	0	0	0	1	1
$\xi_1 + \xi_2$	14	14	1	1	-2	2	-2	2	0	0
$\eta_1 + \eta_2$	12	-12	-1	1	0	0	0	0	2	-2

Then the diagonal of it is

$$\begin{pmatrix} 2184 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -546 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Thus, we obtained K($\mathcal{SL}$(2,13)) = $\oplus\mathbb{Z}_{p}$ for p = 2184,546,1,1,1,2,6,4,,2,1.

3.3 The (CS) of $\mathcal{SL}(2,17)$)

The (RRCT) is :

C_g	1	z	c	zc	a	a^2	a^4	b	b^2	b^3	b^6
$\lvert C_g \rvert$	1	1	144	144	306	306	306	272	272	272	272
$\lvert C_G(g) \rvert$	4896	4896	34	34	16	16	16	18	18	18	18
1_G	1	1	1	1	1	1	1	1	1	1	1
ψ	17	17	0	0	1	1	1	-1	-1	-1	-1
χ_1	72	-72	4	-4	0	0	0	0	0	0	0
χ_2	36	36	2	2	0	0	-4	0	0	0	0
χ_4	18	18	1	1	0	-2	2	0	0	0	0
θ_1	48	-48	-3	3	0	0	0	0	0	-3	3
θ_2	48	48	-3	-3	0	0	0	0	0	3	3
θ_3	16	-16	-1	1	0	0	0	-1	1	2	-2
θ_6	16	16	-1	-1	0	0	0	1	1	-2	-2
$\xi_1 + \xi_2$	18	18	1	1	-2	2	2	0	0	0	0
$\eta_1 + \eta_2$	16	-16	-1	1	0	0	0	2	-2	2	-2

Then the diagonal of it is

$$\begin{pmatrix} 4896 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1224 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 \end{pmatrix}$$

Thus, we obtained K($\mathcal{SL}$(2,17)) = $\oplus\mathbb{Z}_{p}$ for p = 4896,1224,2,1,1,4,2,1,2,3,6.

3.4 The (CS) of $\mathcal{SL}(2,19)$)

The (RRCT) is :

g	1	z	c	zc	a	a^2	a^3	a^6	b	b^2	b^4
$\lvert C_g \rvert$	1	1	180	180	380	380	380	380	342	342	342
$\lvert C_G(g) \rvert$	6840	6840	38	38	18	18	18	18	20	20	20
1_G	1	1	1	1	1	1	1	1	1	1	1
ψ	19	19	0	0	1	1	1	1	-1	-1	-1
χ_1	60	-60	3	-3	0	0	3	-3	0	0	0
χ_2	60	60	3	3	0	0	-3	-3	0	0	0
χ_3	20	-20	1	-1	1	-1	-2	2	0	0	0
χ_6	20	20	1	1	-1	-1	2	2	0	0	0
θ_1	72	-72	-4	4	0	0	0	0	0	-2	2
θ_2	72	72	-4	-4	0	0	0	0	0	2	2
θ_4	18	-18	-1	1	0	0	0	0	0	2	-2
$\xi_1 + \xi_2$	20	-20	1	-1	-2	2	-2	2	0	0	0
$\eta_1 + \eta_2$	18	18	-1	-1	0	0	0	0	2	-2	-2

Then the diagonal of it is

$$\begin{pmatrix} 6840 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1710 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$

Thus, we obtained K($\mathcal{SL}(2,19)$) = $\oplus\mathbb{Z}_{\wp}$ for $\wp = 6840,1710,2,3,1,6,2,1,1,2,4$.

Conflicts Of Interest

There are no conflicts of interest.

Funding

There is no funding for the paper.

Acknowledgment

Our researcher extends his Sincere thanks to the editor and members of the preparatory committee of the Babylonian Journal of mathematics.

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