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Research Article

Comparative Analysis of FTCS, Richardson, and Dufort-Frankel Numerical Methods for Temperature Distribution in a One-Dimensional Rod

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ABSTRACT

This Forward Time Centred Space (FTCS), Richardson, and Dufort-Frankel finite difference methods for solving the one-dimensional heat conduction equation are examined and compared in this paper. The goal is to evaluate each method's accuracy, stability, and computational efficiency by analysing the temperature distribution along a rod. To put the approaches into action, the same starting and ending points for each were used. Not only is FTCS simple and quick to construct, but the results also show that it is conditionally stable. Despite its improved accuracy, Richardson is prone to numerical instability. While the Dufort-Frankel approach is more complicated, it provides better stability. The results help with thermal simulation application requirements-based method selection.

1. INTRODUCTION

One of the most basic issues in thermal engineering is the transfer of heat through solids. Numerical approaches are necessary for complicated situations because analytical solutions are only applicable to simple geometries and boundary conditions. In this paper, three distinct explicit finite difference methods, the FTCS, Richardson, and Dufort-Frankel approaches, are employed to address the one-dimensional heat conduction problem in a rod. Each approach offers a distinct compromise between stability, accuracy, and computing demand. Their usefulness and performance in the real world can be better understood by comparing them. Recognising that the efficacy of numerical methods is contingent exclusively upon their stability [1]. The stability of numerical schemes for time-dependent problems is influenced by false oscillations, particularly at discontinuities. a difference scheme must include the physical domain of dependence of the partial differential equation for convergence [2]. Time-fractional and space-fractional heat equations are the two primary kinds of fractional-order heat equations. The time-fractional version models long-range interactions and non-local heat transfer by including a fractional derivative with respect to time; the space-fractional version models memory effects and anomalous diffusion behaviours by including a fractional derivative in space. In order to resolve space-fractional heat equations, numerous numerical methods have been suggested in published works [3][4]. Karatay I., and Bayramoglu S. [5] conducted a study in which they addressed the numerical solution of the time-fractional heat conduction equation by employing the Crank-Nicolson method. Their research demonstrated how the Crank-Nicolson scheme could be effectively adapted to fractional-order problems, providing accurate and stable temperature profiles over time. Aswin V. S. et al. [6] presented a comparative analysis of three distinct numerical schemes developed to approximate the solution of the convection-diffusion equation, which is commonly encountered in heat and mass transfer problems. By applying these schemes to benchmark problems, they highlighted the advantages and limitations associated with each approach, offering valuable insights for selecting appropriate numerical techniques for solving convection-diffusion equations in practical engineering applications. A et al. [7] investigated the numerical solution of the one-dimensional heat conduction equation using an explicit finite difference scheme. Numerical solutions for time-fractional advection-dispersion equations involving the Riemann-Liouville fractional derivative were obtained directly using the Crank-Nicolson approach in works [8][9]. The numerical analysis is important for many engineering applications, as it can make it easier to simulate and analyze. Improving the thermal performance of flat plate solar collectors is closely related to understanding the temperature distribution in a one-

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dimensional rod using numerical methods like FTCS, Richardson, and Dufort-Frankel. This is especially important in regions like Iraq, where solar energy is abundant. To maximise efficiency and power capture, precise simulations of heat transfer within a solar collector's absorber plate are required [10], [11], [12].
Examining the accuracy, stability, and computing efficiency of three numerical methods, FTCS, Richardson, and Dufort-Frankel, in solving the one-dimensional heat conduction equation is the main objective of this study. Using consistent initial and boundary circumstances, the study aims to apply various methodologies and determine their strengths and limits. One of the main goals is to see which methods work best for representing the heat dispersion in thermally conductive materials, as the absorber plates of Iraqi flat plate solar collectors. In the context of solar energy applications and heat transfer system design, the findings are meant to offer researchers and engineers practical advice on choosing the right numerical schemes for thermal simulations.

2. SYSTEM DESCRIPTION

In the model, the border temperatures of the rod were kept constant. The domain was first subjected to a temperature distribution. In MATLAB, the simulations were executed with a constant time step and equal grid spacing. A 10-centimeter-long, one-dimensional aluminium rod is the system under consideration here. The goal is to use numerical methods to examine the temperature distribution along the rod as it changes over time. The material's thermal conductivity, as measured in degrees Celsius, is $\mathrm{k} = 0.49\ \mathrm{cal/s.cm.°C}$.
The spatial and temporal discretization steps are defined as:

- Spatial step $\Delta x = 2\mathrm{cm}$
- Time step $\Delta t = 0.1\mathrm{s}$

The boundary conditions are fixed for all time steps as:

- T(0 cm) = 100 ℃
- T(10 cm) = 50 ℃

The material properties for aluminum used in the simulation are:

- Specific heat capacity $\mathrm{C} = 0.2174\ \mathrm{cal/g.°C}$
- Density $\rho = 2.7\ \mathrm{g}/cm^3$

3. NUMERICAL ANALYSIS

3.1. FTCS Method

The forward difference and centred difference methods are used to discretise the time derivative and spatial derivative, respectively, in the FTCS method:

$$T_i^{n+1} = \mathrm{T}_i^n + r(\mathrm{T}_{i+1}^n - 2\mathrm{T}_i^n + \mathrm{T}_{i-1}^n) \quad (1)$$

$$r = \alpha \frac{\Delta t}{\Delta x^2} \quad (2)$$

In this context, T is temperature, t is time, x is space, and α is diffusion.

3.2. Richardson Method

The Richardson technique makes use of a space-and time-centered difference

$$T_i^{n+1} = \mathrm{T}_i^{n-1} + 2r(\mathrm{T}_{i+1}^n - 2\mathrm{T}_i^n + \mathrm{T}_{i-1}^n) \quad (3)$$

In the given context, the symbols $\dot{\mathrm{E}}\mathrm{x}_{\mathrm{W,in}}$ is the exergy of the input water, $\dot{\mathrm{E}}\mathrm{x}_{\mathrm{W,out}}$ is the exergy of the output water, and $\dot{\mathrm{E}}\mathrm{x}_{\mathrm{Solar,in}}$ is the input exergy rate of solar.

3.3. Dufort-Frankel Method

Using values from two time levels, this approach alters the FTCS scheme to improve stability.

$$T_i^{n+1} = \frac{(1-2r)T_i^{n-1} + 2r(\mathrm{T}_{i+1}^n + \mathrm{T}_{i-1}^n)}{1+2r} \quad (4)$$

4. RESULTS AND DISCUSSION

Temperature distributions show that all methods predict similar trends, but differ in numerical behavior. FTCS is prone to instability at higher time steps. Richardson offers better accuracy but shows oscillations under improper conditions. Dufort-Frankel maintains stability with larger time steps, though with higher computational cost. Error analysis confirmed that Dufort-Frankel balances accuracy and stability effectively. Despite its apparent simplicity and ease of implementation, the FTCS technique demonstrated conditional stability. The method's instability became more pronounced at bigger time step sizes, leading to non-physical or oscillatory temperature readings, especially close to the rod's centre nodes. Long simulations take more time to compute since this instability limits the permissible time step to small numbers. The Richardson approach, on the other hand, used centered differences in space and time to achieve better accuracy. In the transitory zone in particular, the symmetrical discretisation approach recorded more accurately the small fluctuations in the progression of temperature. Although the precision was improved, numerical stability was diminished. Inappropriate selection of time step sizes led to oscillations, suggesting that stability criteria are more important when using Richardson's approach. To guarantee accurate findings, a tighter regulation of discretisation settings is required. When considering stability, the Dufort-Frankel technique was exceptional. Greater time steps could be used without experiencing instability, as the formulation incorporates values from both the previous and current time levels. Because of this, it can be used for simulations with lengthy time horizons or in situations where computing performance is crucial. The method's implicit nature and data handling requirements make it computationally demanding and somewhat more complicated to implement, but it consistently produces physically realistic temperature profiles across the domain that are smooth and realistic. The results of the error analysis showed that, out of the three methods, the Dufort-Frankel one strikes the best compromise between stability and accuracy. It eschews both the rigid constraints of FTCS and Richardson's oscillatory tendencies while keeping in close accord with the predicted theoretical behaviour. This makes it an ideal material for thermal energy storage systems and solar absorber panels, two real-world engineering uses of transient heat conduction.

TABLE I. FORWARD TIME CENTRED SPACE METHOD

t (s)	**T (0 cm)**	**T (2 cm)**	**T (4 cm)**	**T (6 cm)**	**T (8 cm)**	**T (10 cm)**
0	100	0	0	0	0	50
0.1	100	2.0875	0	0	1.0437	50
0.2	100	4.0878	0.0436	0.0218	2.0439	50
0.3	100	6.0056	0.1275	0.0645	3.0028	50
0.4	100	7.8450	0.2489	0.1271	3.9225	50
0.5	100	9.6102	0.4050	0.2089	4.8052	50
0.6	100	11.3049	0.5930	0.3089	5.6527	50
0.7	100	12.9328	0.8107	0.4264	6.4669	50
0.8	100	14.4973	1.0557	0.5605	7.2495	50
0.9	100	16.0016	1.3260	0.7105	8.0023	50
1	100	17.4487	1.6195	0.8756	8.7268	50
1.1	100	18.8415	1.9344	1.0550	9.4245	50
1.2	100	20.1828	2.2690	1.2481	10.0968	50
1.3	100	21.4750	2.6216	1.4541	10.7450	50
1.4	100	22.7206	2.9908	1.6724	11.3705	50

TABLE II. RICHARDSON METHOD

t (s)	T (0 cm)	T (2 cm)	T (4 cm)	T (6 cm)	T (8 cm)	T (10 cm)
0	100	0	0	0	0	50
0.1	100	2.0875	0	0	1.0437	50
0.2	100	4.0007	0.0872	0.0436	2.0003	50
0.3	100	5.9321	0.1616	0.0835	2.9660	50
0.4	100	7.6871	0.3248	0.1672	3.8437	50
0.5	100	9.4788	0.4624	0.2436	4.7396	50
0.6	100	11.0899	0.6921	0.3640	5.5456	50
0.7	100	12.7567	0.8828	0.4736	6.3792	50
0.8	100	14.2366	1.1708	0.6277	7.1202	50
0.9	100	15.7918	1.4056	0.7674	7.8984	50
1	100	17.1517	1.7447	0.9520	8.5802	50
1.1	100	18.6075	2.0157	1.1189	9.3092	50
1.2	100	19.8571	2.4000	1.3314	9.9371	50
1.3	100	21.2246	2.7000	1.5228	10.6225	50
1.4	100	22.3726	3.1243	1.7605	11.2012	50

TABLE III. DUFORT-FRANKEL FINITE DIFFERENCE METHOD

t (s)	T (0 cm)	T (2 cm)	T (4 cm)	T (6 cm)	T (8 cm)	T (10 cm)
0	100	0	0	0	0	50
0.1	100	2.0875	0	0	1.0437	50
0.2	100	4.1750	0.0872	0.0436	2.0875	50
0.3	100	6.1790	0.1761	0.0908	3.0895	50
0.4	100	8.1830	0.3453	0.1781	4.0916	50
0.5	100	10.1104	0.5179	0.2722	5.0554	50
0.6	100	12.0380	0.7643	0.4033	6.0197	50
0.7	100	13.8952	1.0157	0.5441	6.9487	50
0.8	100	15.7528	1.3353	0.7190	7.8786	50
0.9	100	17.5459	1.6610	0.9061	8.7761	50
1	100	19.3395	2.0499	1.1247	9.6750	50
1.1	100	21.0739	2.4460	1.3578	10.5442	50
1.2	100	22.8092	2.9008	1.6201	11.4152	50
1.3	100	24.4902	3.3638	1.8988	12.2591	50
1.4	100	26.1724	3.8815	2.2047	13.1054	50

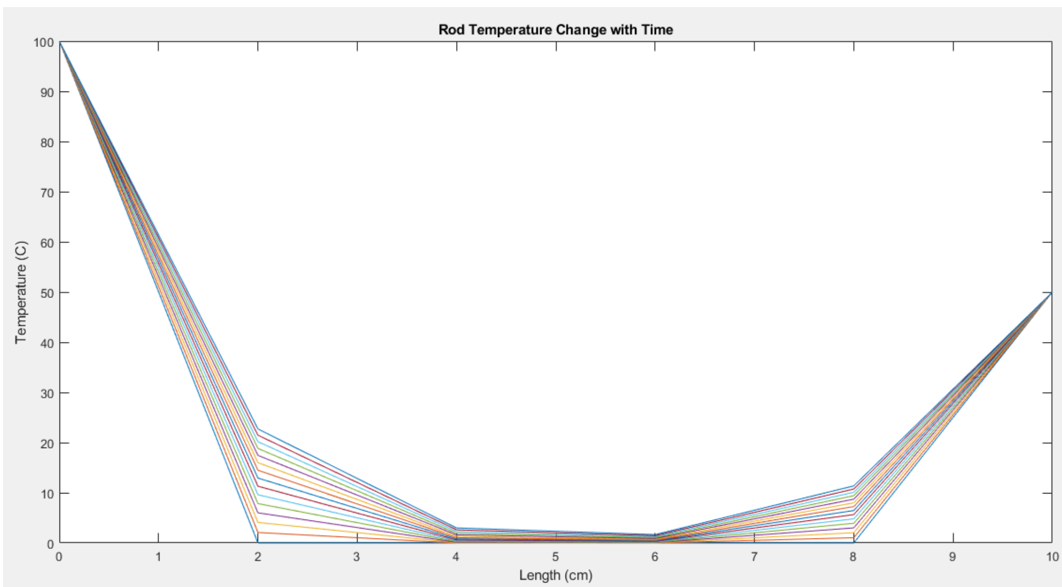

Fig. 1. Forward time centred space method.

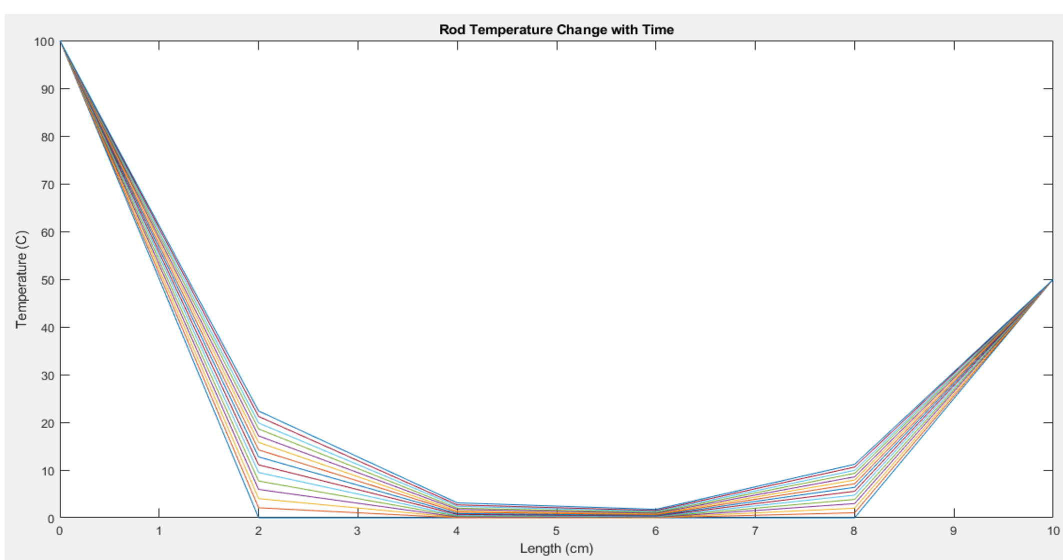

Fig. 2. Richardson method.

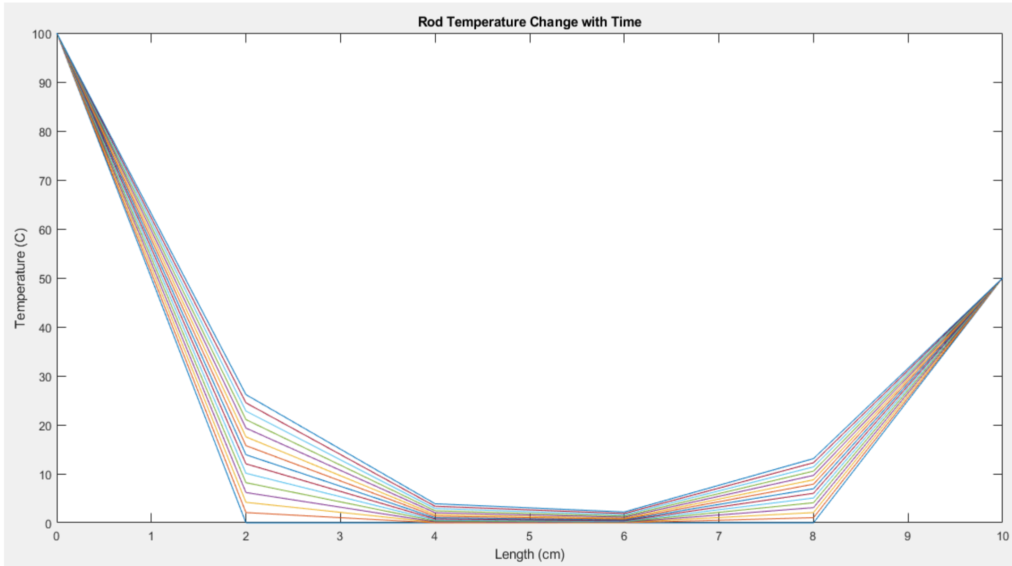

Fig. 3. Dufort-Frankel finite difference method.

5. CONCLUSIONS

Every approach has its own set of benefits and drawbacks. Fast time-dependent steady-state simulations are well suited to FTCS. Although it demands meticulous parameter tweaking, Richardson is beneficial for situations requiring great accuracy. The substantial stability of Dufort-Frankel for bigger time steps makes it a perfect fit for simulations with lengthy durations. The comparison clearly shows that while choosing a numerical approach, the problem's requirements, including stability limitations, needed precision, and available computer resources, should be considered. This study highlights the importance of more robust schemes, such as Dufort-Frankel, for practical engineering applications that require stability over long simulation periods, as opposed to simpler schemes like FTCS, which are better suited for training and preliminary analysis. Researchers and engineers can use this thorough review as a roadmap to select the best thermal analysis method for one-dimensional domains.

Conflicts Of Interest

The author's paper explicitly states that there are no conflicts of interest to be disclosed.

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